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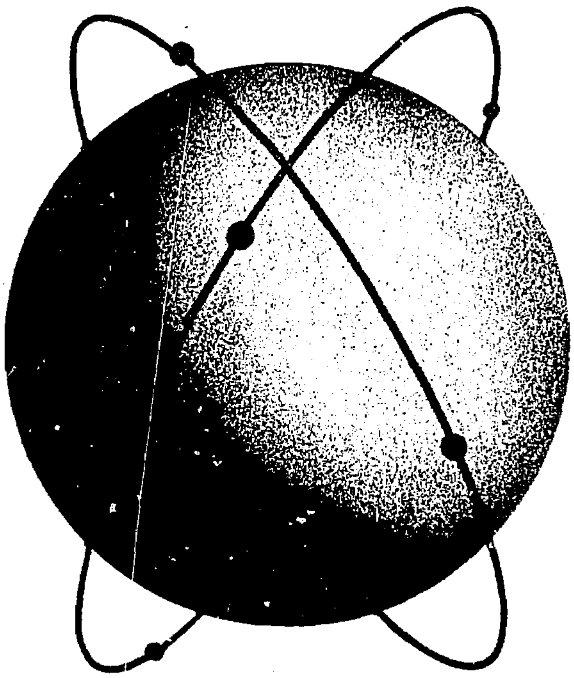
ABSTRACT

As the fourth lesson of the Articulated Multimedia Physics Course, instructional materials are presented in this study guide. The subject matter is concerned with displacements, speeds, scalers, vector sum, and vector analysis. The content is arranged in scrambled form, and the use of matrix transparencies is required for students to control their learning progression. Students are asked to use magnetic tape playback, instructional tapes, and single concept films at the appropriate place in conjunction with a worksheet. Included are a problem assignment sheet, a study guide slipsheet, and illustrations for explanation purposes. Related documents are SE 015 963 through SE 015 977. (CC)

ARTICULATED MULTIMEDIA PHYSICS



LESSON 4

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

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Old Westbury, Long Island

New York, N.Y.

ARTICULATED MULTIMEDIA PHYSICS

Lesson Number 4

VECTORS

IMPORTANT: Your attention is again called to the fact that this is not an ordinary book. It's pages are scrambled in such a way that it cannot be read or studied by turning the pages in the ordinary sequence. To serve properly as the guiding element in the Articulated Multimedia Physics Course, this Study Guide must be used in conjunction with a Program Control equipped with the appropriate matrix transparency for this Lesson. In addition, every Lesson requires the availability of a magnetic tape playback and the appropriate cartridge of instructional tape to be used, as signaled by the Study Guide, in conjunction with the Worksheets that appear in the blue appendix section at the end of the book. Many of the lesson Study Guides also call for viewing a single concept film at an indicated place in the work. These films are individually viewed by the student using a special projector and screen; arrangements are made and instructions are given for synchronizing the tape playback and the film in each case.

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New York Institute of Technology
Articulated Multimedia Physics

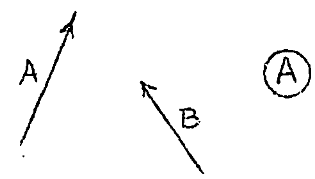
LESSON 4

STUDY GUIDE SLIP SHEET

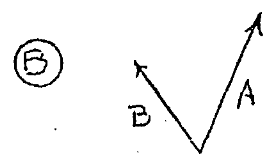
STUDY GUIDE TEXT: Page 54, middle of page. Right after "It always works" add the following: "See slip sheet for a supplementary notebook entry on vector subtraction." When you have done this, copy the following into your notebook, entitling it "Supplementary Note", exactly as shown

Supplementary Note

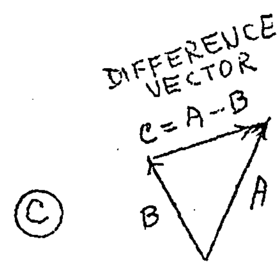
A. Suppose you are given two vectors, $\vec{A}$ and $\vec{B}$ as in Figure A, and you are told to subtract $\vec{B}$ from $\vec{A}$, that is, solve for $\vec{A} - \vec{B}$.



B. To use the short-cut method of vector subtraction, join $\vec{A}$ to $\vec{B}$, tail-to-tail without revers- any directions at all. See Figure B. Both vectors are to be drawn to scale, of course.



C. Complete the triangle by drawing the resultant vector from the head of $\vec{B}$ to the head of $\vec{A}$, plac- the new arrowhead on this difference vector where it joins $\vec{A}$. In other words, the direction of the difference vector is toward the vector which is the first term in the subtraction statement: $\vec{A} - \vec{B} = \vec{C}$. The first term in this statement is the vector $\vec{A}$, of course. See Diagram C.



STUDY GUIDE DIAGRAMS: Page 103, Figure 12. This diagram is drawn to half-scale and may be confusing. Therefore, write alongside this diagram, "Refer to full-scale diagram on the slip sheet for this Lesson." The full-scale diagram will be easier for you to interpret and is given for your convenience on the next page of this slipsheet.

(next page, please)

Physics has made vast contributions to man's understanding and control of bodies in motion. Our ability to navigate on land, at sea, and in the air is based upon an understanding of motion and of how to describe it scientifically. From the launching of our first artificial satellite to our final conquest of interplanetary space, our endeavors have been and will continue to be governed by considerations in which the motion of objects plays the leading role.

One of the most important aspects of motion is the subject of this lesson. To illustrate the need for this study, suppose you take a hypothetical ride in a car along a well laid out route such as that shown in Figure 1.

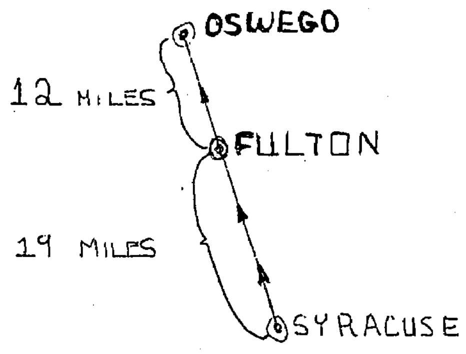


Figure 1

Between the cities of Syracuse and Oswego, N. Y., you will drive a distance of 31 miles on a road which is nearly straight. As you drive from Syracuse to Oswego, you will pass through the town of Fulton, which is 19 miles from Syracuse and 12 miles from Oswego.

Please go on to page 2.

Imagine that you have just completed the trip from Syracuse to Oswego. Suppose that someone then asked you, "What was the length of your trip?" You could answer without hesitation that the length of the trip was 31 miles. If you should then be asked, "How far are you from your starting point?" you could again immediately answer 31 miles. Thus, in this very simple kind of motion, the length of the trip and the distance from your starting point are exactly the same.

Next assume that you finish your business in Oswego and start back toward Syracuse, stopping at Fulton to make a purchase. In response to the storekeeper's query, "What was the length of the trip you made today?" you would do a bit of mental arithmetic (31 miles plus 12 miles) and answer that 43 miles was the length of your trip. (Refer to Figure 2.)

Can you see what's coming next? If he now asked, "How far are you from your starting point?" you would answer 19 miles.

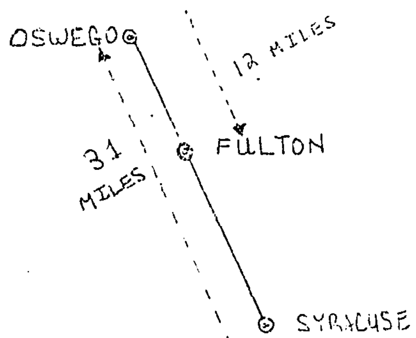


Figure 2

You could have obtained this figure either by knowing the distance between Syracuse and Fulton or by subtracting 12 miles from 31 miles. The point is that you can no longer give the same answer to the two questions: "What was the length of your trip?" and "How far are you from your starting point?"

Please go on to page 3.

To make our discussion a bit easier, we are going to call the distance from the starting point by the name of displacement. We shall continue to call the length of a trip "trip length" or just "Length." In the Syracuse-Oswego example, there is certainly no mystery as to why displacement and length are the same in one case and different in the other. As long as you continue to move in the same direction in a straight line, displacement and length are equal; the moment your direction changes, displacement and length become different.

So, you can see that we are dealing with two very different quantities when we speak of length of trip and displacement. If you were to travel around a perfectly square park 5 miles on each side, starting from point A in Figure 3, the trip length would obviously be 20 miles, but your displacement at the end of the trip would be zero! Note that the calculation of the perimeter of the park, or the trip length, is done without any attention to direction of motion at all. But the determination of displacement must take direction into account.

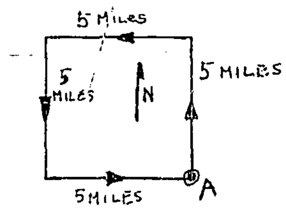


Figure 3

Please go on to page 4.

In some physical measurements, direction is never important. Such quantities are called <u>scalar</u> quantities. Trip length, temperature, and mass are all scalar quantities. In many cases, however, knowing the direction taken by the measured quantity is absolutely essential in order to describe the quantity fully. Such quantities are called <u>vector</u> quantities. Clearly, <u>displacement</u> must be a vector quantity. Other vector quantities are velocity, force, and momentum.

In this lesson you will learn how to think about vectors, how to write them, and how to manipulate them.

<u>NOTEBOOK ENTRY</u>

Lesson 4

1. (a) A <u>scalar</u> quantity is a quantity that may be fully described by stating its <u>magnitude</u> (size or "bigness").
   (b) A <u>vector</u> quantity may be fully described only if <u>both its magnitude and direction</u> are stated.
   (c) Examples: <u>scalars:</u> length, mass, temperature
   <u>vectors:</u> displacement, force, velocity

Before continuing, please turn to page 141 in the blue appendix.

A perfectly rectangular plot of land (Figure 4) measures 30.0 m in length and 15.0 m in width. Suppose you start from point A and walk around the perimeter of the plot in the direction indicated by the arrows. After having walked from A to B to C and then to D, the length of your trip would have been 60.0 m.

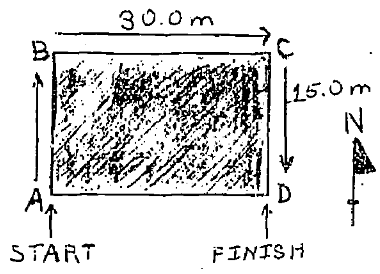


Figure 4

Which of the following best describes your displacement?

(1)

A 60.0 m to the north, to the east, and then to the south.

B 30.0 m to the north, to the east, and then to the south.

C 30.0 m to the east.

YOUR ANSWER --- A

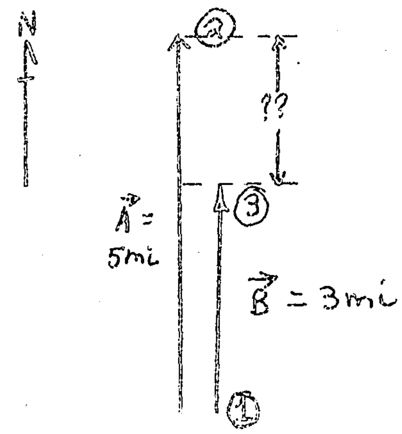


Figure 27

We have brought the original vectors closer to each other in Figure 27. This should make it clear to you that point (3) is not north of point (2). Check the compass directions. In what direction is point (3) relative to point (2)? We agree that point (3) is north of point (1) but this was not the question.

Please return to page 126 and select another answer.

YOUR ANSWER --- A

This answer is not right. We haven't used the scale improperly. As you see, the scale is: 1 cm = 1 mi. The northerly trip as shown by A is 5 mi and the vector is 5 cm long. The trip off at some angle to the northwest is 3 mi in length and we have shown this vector as 3 cm in length. Since we have not yet discussed the magnitude of the line segment from (2) to (3), we haven't violated any scale rules at all.

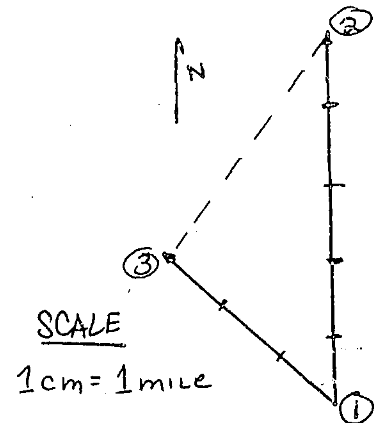


Figure 29

Please return to page 57 and select another answer.

YOUR ANSWER --- A

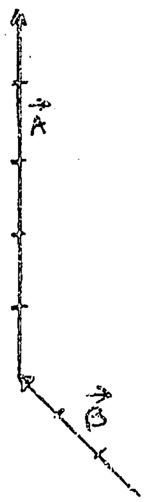


Figure 30

The diagram in Figure 30 is not right. $\vec{B}$ has been shifted, but its direction has not been reversed.

In the original problem diagram, $\vec{B}$ was directed toward the northwest. In Figure 30, where $\vec{B}$ is supposed to be reversed in direction, this vector is still pointing in a general northwesterly direction.

Please return to page 127 and select another answer.

YOUR ANSWER --- B

This answer is false. In vector addition, the line segments may be shifted parallel to themselves to make the required head-to-tail connection, but they may <u>never</u> be reversed in direction. Such reversal signifies subtraction, not addition.

Now refer to diagram (1) and (3). Note that when $\vec{B}$ was moved from its initial position in diagram (1) to its new position in diagram (3), <u>it was reversed in direction</u>, hence the operation cannot be addition. Thus, if vector C is not the sum of $\vec{A}$ and $\vec{B}$, what is it?

(1) (3)

Figure 38

Please turn to page 67 to check your answer.

YOUR ANSWER --- A

There's just one thing wrong with this answer. You recognized that the trip length is a scalar quantity, hence no direction can be specified. You also recognized that the displacement is a vector quantity and that direction must be given. However, you have the direction wrong. Look again at the diagram. Displacement is measured along the straight line AC. Since the ant wound up at position C, he is not to the east of his starting point, is he?

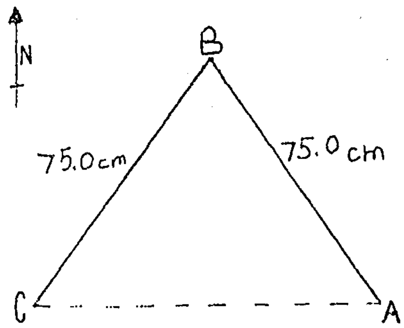


Figure 5

Always specify direction for a vector quantity from the point of view of an observer at the starting point. If you were the observer at point A, in which direction would you be looking to spot the ant at his final position? This is the direction of the displacement vector.

Please return to page 44 and select another answer.

YOUR ANSWER --- B

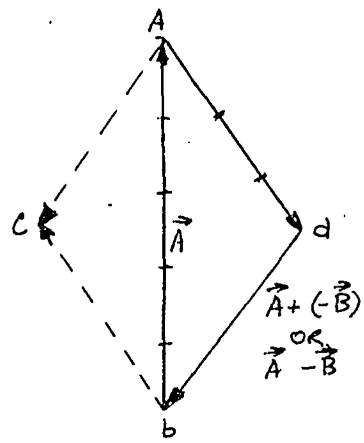


Figure 33

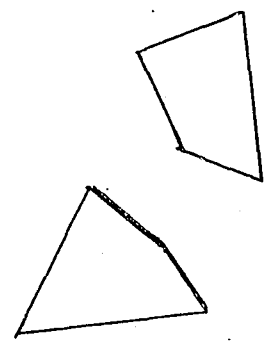


Figure 34

There is no such theorem! Referring to Figure 34, you see two polygons, neither of which have equal or parallel sides. Only in a parallelogram are opposite sides equal and parallel.

Please return to page 122 and select another answer.

YOUR ANSWER --- C

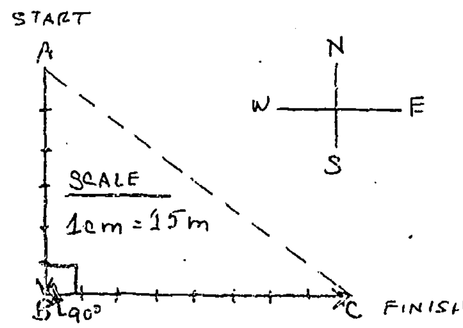


Figure 12

You probably failed to grasp the meaning of total displacement. On any series of trips, in any directions, the total displacement at the end may be determined by measuring the distance between the starting point and finishing point along a straight line connecting the two. In our example, the boy made two trips. The first was from A to B and the second was from B to C. He finishes at point C, so we draw a straight line (AC) from his starting position to point C. This line, AC, is his total displacement.

We next measure off line AC in centimeters and find it 10 cm long. Our scale informs us that each centimeter is worth 15 meters, so we may write the scale as 15 m/cm. To find the magnitude of the displacement, we merely multiply the scale value by the total length of the vector AC in centimeters. Thus:

15 m/cm x 10 cm

Please return to page 103 and select an answer.

YOUR ANSWER --- B

You are correct. The trip length is a scalar, so no direction is expressed or implied. The displacement is a vector quantity and to be fully described, must include a statement of direction. The direction of the displacement vector is that of a straight line joining the starting position and the finishing position, in the direction of the finishing position.

There is a simple conventional way of representing vector magnitude and direction in physics. Vectors are represented by straight-line segments with arrowheads to show direction. The length of the line segment indicates the magnitude of the vector quantity, while the direction of the arrow shows the direction of the vector. (See Figure 6.)

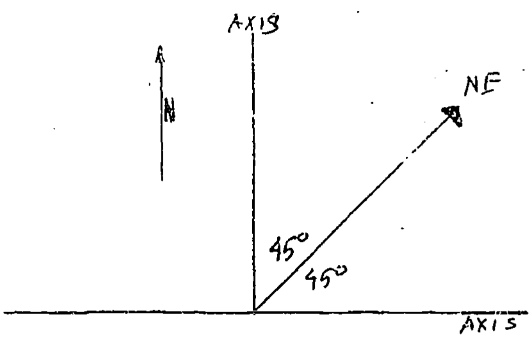


Figure 6

For the moment, let us concern ourselves only with direction. In the example shown in the drawing the vector is shown as a straight line segment. Remember that in algebra angle measurements always start in the first quadrant along the X-axis. The direction of this vector might therefore be specified in one of two ways:

(1) Its direction might be stated as being exactly to the northeast.
(2) Its direction might be stated as being:

(3)

A approximately northeast.

B 45° to a north-south axis drawn through the starting point.

C 45° to an east-west axis drawn through the starting point.

YOUR ANSWER --- A

You are correct. In this 3:4:5 triangle, the sides must be in the ratio 24:32:40; hence the hypotenuse (or displacement) must be 40 km.

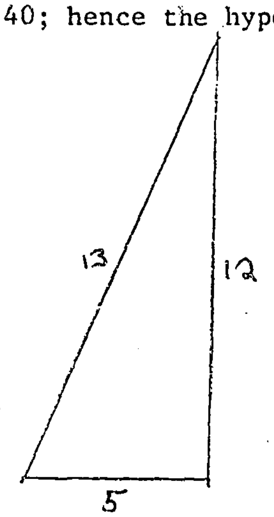


Figure 16

There is another special triangle that is often encountered in physics problems dealing with vectors. Refer to Figure 16. The triangle in this case is a 5:12:13 triangle. You should check this ratio with the Pythagorean theorem to satisfy yourself that it is quite correct.

Suppose we walk 20 feet east, then 48 feet north. Will the vectors representing this situation form a 5:12:13 triangle? Yes, they will. If you divide 20/5, you get 4; if you divide 48/12, you again get 4. Hence, 20:48 = 5:12. This means that the hypotenuse, representing the total displacement, must be how many feet long?

(12)

A The hypotenuse of a triangle having legs of 20 ft and 48 ft, respectively, must be 52 ft.

B The hypotenuse of a triangle having legs of 20 ft and 48 ft, respectively, must be 56 ft.

YOUR ANSWER --- B

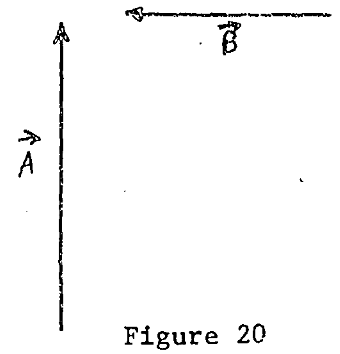


Figure 20

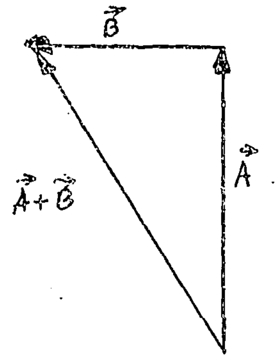


Figure 22

You are correct. In this vector shift, $\overline{B}$ has been moved parallel to itself, downward and to the left, so that its tail is joined to the head of $\overline{A}$. In doing this, no alteration has been made in the direction or the magnitude of either vector. Good work.

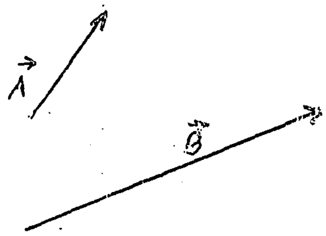


Figure 24

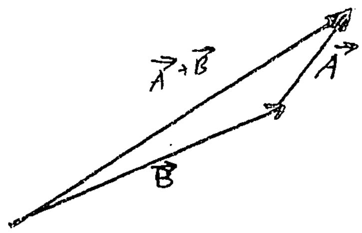


Figure 25

Now examine Figures 24 and 25. The latter is presumed to be the final picture showing how the vectors in Figure 24 have been shifted to put them in a form suitable for obtaining the vector sum. Now answer the question below.

Which one of the following statements is true?

(15)

A Figure 25 is incorrect because the magnitude of one of the vectors has been changed.

B Figure 25 is incorrect because both vectors have been moved, instead of just one.

C Figure 25 is a correct representation.

D Figure 25 is incorrect because neither vector has been moved parallel to itself.

YOUR ANSWER --- C

Not at all. The atmosphere turns with the Earth as it rotates on its axis, so that the relative motion of the air on a windless day is zero with respect to the earth.

Think of it this way. As the Earth rotates on its axis, the ground at the equator moves at a speed of very nearly 1,000 miles per hour with respect to some fixed point in space, like the sun. Suppose that the atmosphere were not dragged around with the Earth as it spins. Then if you were standing motionless on any point along the equator you would be moving 1,000 miles per hour relative to the still air. You would then be subjected to the buffeting of a 1,000 mi/hr hurricane constantly. You know this doesn't happen at the equator or any other spot on the Earth; hence the atmosphere must be moving at very nearly the same speed, and in the same direction, as the Earth.

Atmospheric rotation is the product of two factors. First, the gravitational attraction of the Earth for air molecules pulls them downward into contact with the surface. Second, frictional drag then pulls the atmosphere around to follow the surface as the Earth rotates.

Hence, we do not have to take into account the rotation of the Earth in solving airplane ground-speed problems.

Please return to page 128 and select another answer.

YOUR ANSWER --- B

You are correct. Merely giving the path as a "diagonal" does not specify direction. Since velocity is a vector quantity the statement does not describe velocity. Rather, it gives the boy's speed, or the magnitude of his velocity.

Before continuing, please turn to page 144 in the blue appendix.

We'll pause a moment for a brief notebook entry and a notebook check.

NOTEBOOK ENTRY

4. Velocity and Speed

(a) Velocity is defined as the rate of change of displacement, or displacement per unit time. Since displacement is a vector quantity, then velocity is also a vector quantity. Thus, both direction and magnitude must be stated in describing the velocity of any moving object.

(b) Speed is the term used to describe how fast something is moving relative to some reference point. No direction is implied when speed is stated. Speed is the magnitude of a given velocity. For example, if we say that a man walks 8 ft per sec due northwest, we are giving his velocity. His speed is, therefore, 8 ft per sec.

NOTEBOOK CHECK

Which item in Notebook Entry 3 tells you to rewrite $\vec{A} - \vec{B}$ as $\vec{A} + (-\vec{B})$ in vector subtraction?

(24)

A Item 3(d).

B Item 3(c).

C Item 3(b).

D Item 3(a).

YOUR ANSWER --- B

You are correct. This is the best choice of the scales listed. It is not the only scale you might use, but it will give you a nice large diagram well within the confines of a standard sheet of paper.

Using this scale, set up your vector diagram to find the resultant velocity. Measure carefully. We shall want you to find the speed and the resultant direction of the plane in terms of the angle it makes with the west-to-east vector. Measure this angle to the nearest degree with a reasonably good protractor. When you have finished, select from the answers below the one that most closely approximates yours. (The speed should be given to 3 significant figures.)

(26)

A 210 km/hr, 60° N of E.

B 227 km/hr, 70° N of E.

C 207 km/hr, 76° N of E.

D My answer is different from any of these.

YOUR ANSWER --- B

(A) (B)

Figure 41

Why not? The vectors $\vec{v}_a$ and $\vec{v}_b$ are correctly summed up since they are head-to-tail just as they should be joined. When you add a pair of components correctly, you are bound to come out with their resultant.

Please return to page 92 and select the correct answer.

YOUR ANSWER --- B

Your choice of this answer indicates that you are confusing the number of significant figures in the vector lengths measured in centimeters with the actual trip lengths measured in kilometers. Although the line segments are measured to 2 significant figures, both of the original trip lengths are given to 3 significant figures; hence the resultant displacement may be written to 3 significant figures.

Please return to page 104 and select the correct answer.

YOUR ANSWER --- C

Not a good guess. Rule 2(b) states: The displacement resulting from two trips in opposite directions along a straight line is the algebraic sum of the two trip lengths. Arbitrary signs are given to each trip. (+) for one and (-) for the other since they are in opposite directions.

Please return to page 79 and select another answer.

YOUR ANSWER --- C

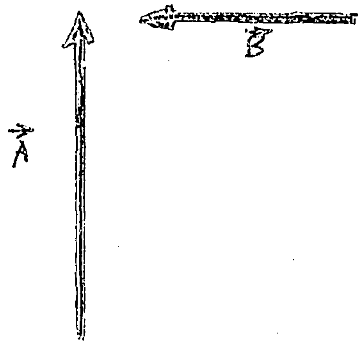


Figure 20

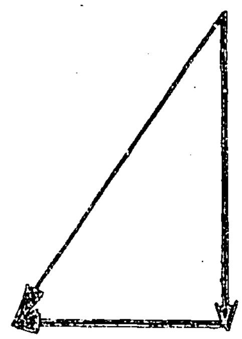


Figure 23

It shows the proper combination of the vectors in Figure 20 for the purpose of obtaining the vector sum.

The principle which describes the correct method of shifting vectors for the purpose of obtaining their sum states that either vector may be moved parallel to itself until the two are joined head-to-tail.

In your diagram choice, $\vec{B}$ has been moved parallel to itself in conformity with the first part of this principle. And, in the final position $\vec{A}$ and $\vec{B}$ are joined head-to-tail, again in accordance with the principle. Notwithstanding this, <u>the diagram is still wrong.</u>

Examine the two original vectors in Figure 20 carefully. Now do the same for Figure 23. Note particularly the directions of $\vec{A}$ and $\vec{B}$ in both diagrams. What did you do that was wrong?

Please turn to page 66.

YOUR ANSWER --- C

Incorrect. We have already stated that AC is the actual path taken by the swimmer as a result of the combination of his own velocity through the water and the velocity of the stream. AC is not the vector showing the direction of the water velocity alone.

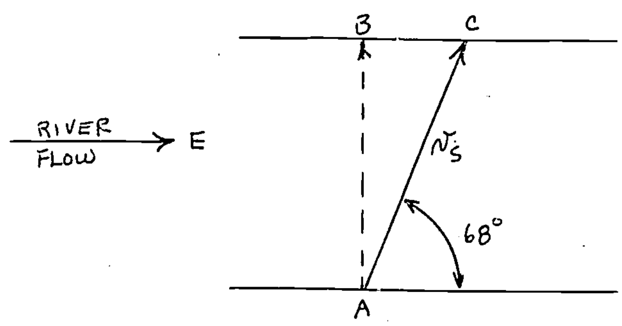


Figure 47

Please return to page 93 and select another answer.

YOUR ANSWER --- A

You might say that this answer has 1½ errors! Since $\vec{U}$ is to be subtracted from $\vec{V}$, the $\vec{U}$ must come after the minus sign, not before it. This is a definite error.

You could write it this way: $\vec{V} - \vec{U}$

Now the order is correct; but this is not the best way to show a vector subtraction. You should not indicate the vector operation with a minus sign at all, as we have been emphasizing. Rather, you should reverse the sign of the $\vec{U}$ and then specify the operation as addition.

Please return to page 135 and select another answer.

YOUR ANSWER --- B

You are correct. Figure 42 shows proper vector additions throughout. Therefore, all of these pairs of original velocities ($v_a + v_b$, $v_c + v_d$, $v_e + v_f$, and $v_g + v_h$) are possible components of the resultant v_r.

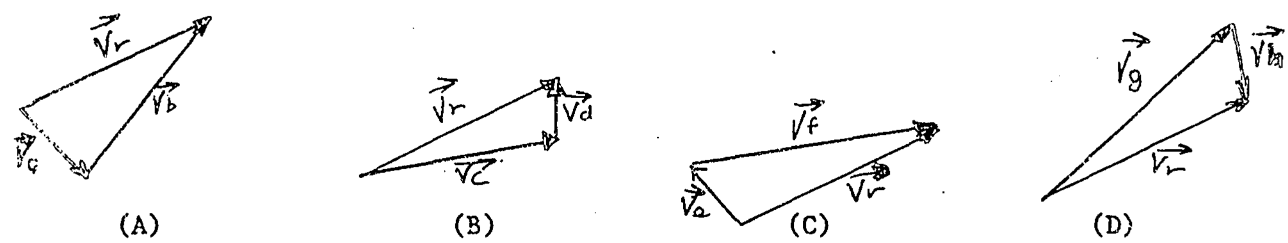


Figure 42

The pairs of velocities in the examples of Figure 42 were chosen more or less at random. The only basis for selection was that each pair of velocities, when joined head-to-tail correctly, would call for the same velocity, v_r, to close the triangle.

Judging from what you have just learned, which of the following statements would you consider the only <u>true</u> one?

(29)

A Any vector may be shown to be the resultant of only <u>one</u> distinct pair of components.

B Any vector may be shown to be the resultant of <u>four</u>, and only four, pairs of components.

C Any vector may be shown to be the resultant of an <u>infinite</u> number of possible pairs of components.

YOUR ANSWER --- A

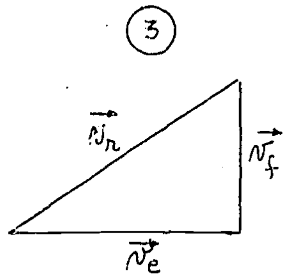


Figure 45

You are correct. Since v_e is perpendicular to v_f, and since v_e is horizontal, then v_f must be vertical. Thus, v_e and v_f are the only two possible components that meet the conditions of the problem. Note that, in constructing the correct diagram, v_e was drawn horizontally and made just long enough to allow v_f to be erected as a perpendicular to it so that the head of v_f could then join the head of v_r.

We will now investigate a practical situation involving the technique of resolution of vectors. A smooth ski slope makes an angle of 30.0° with level ground and is 5.00 km long. A skier is towed from the bottom to the top along the slope.

(a) How far did he travel horizontally?

(b) How high is the top of the slope above ground level?

The diagram comes first, of course; see Figure 46. The actual trip up the slope is symbolized by s, drawn at an angle of 30.0° to the ground. Using the scale 1 cm = 0.5 km, what should be the length of vector s?

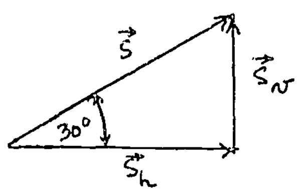


Figure 46

Please turn to page 129 for the right answer.

YOUR ANSWER --- C

You are correct. You are definitely learning how to handle vectors. Like so many ideas and concepts in physics, vectors appear and reappear in many phases of our subject. You will have many opportunities to use them when you reach the study of forces.

Before doing anything else at this time, please turn to page 145 in the blue appendix.

Now that you have completed the Worksheet on page 145, please go on to page 28.

You have now completed the study portion of Lesson 4 and your Study Guide Computer Card and A V Computer Card should be properly punched in accordance with your performance in this Lesson.

You should now proceed to complete your homework reading and problem assignment. The problem solutions must be clearly written out on 8½" x 11" ruled, white paper, and then submitted with your name, date, and identification number. Your instructor will grade your problem work in terms of an objective preselected scale on a Problem Evaluation Computer Card and add this result to your computer profile.

You are eligible for the Post Test for this Lesson only after your homework problem solutions have been submitted. You may then request the Post Test which is to be answered on a Post Test Computer Card.

Upon completion of the Post Test, you may prepare for the next Lesson by requesting the appropriate:

1. study guide
2. program control matrix
3. set of computer cards for the lesson
4. audio tape

If films or other visual aids are needed for this lesson, you will be so informed when you reach the point where they are required. Requisition these aids as you reach them.

Good Luck!

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YOUR ANSWER --- A

You are correct. Writing it this way, that is, using the suggested sign convention, makes for good vector habits. Keep it up.

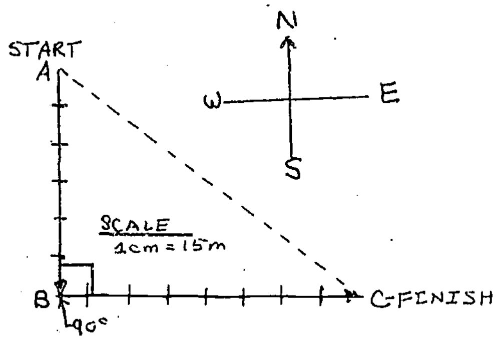


Figure 12

We next investigate the method of finding displacement when the two trips take place at right angles to each other, as in Figure 12. A boy starts walking south from point A and proceeds for a distance of 90 m to point B. We choose a scale:

$$1 \text{ cm} = 15 \text{ m}. \quad \text{Since } \frac{90 \text{ m}}{15 \text{ m/cm}} = 6 \text{ cm}.$$

We make the vector line segment 6 cm long, directed toward the south. This is vector AB. He then walks due east as shown by vector BC. According to selected scale, how far due east did he walk?

(7)

A He walked 8 cm due east.

B He walked 120 m due east.

YOUR ANSWER --- A

You are correct. A headwind, or wind blowing opposite to the plane's flight direction, would slow down the plane relative to the ground. A tailwind would increase the plane's ground speed. A cross wind would tend to make the plane drift sideward. Thus, we need to know the wind direction before we can solve the airplane problem.

Whenever the meaning of a measured quantity involves direction, we know we are speaking of a vector.

Velocity is a vector quantity. We cannot describe the velocity of an airplane, an automobile, or a pedestrian without stating both magnitude and direction. The numerical part of a velocity statement is called the speed of the object. From now on, when we mean to speak of the vector we shall use the word "velocity." The "speed" of an object tells how fast the object travels relative to some reference point, but it does not give the direction, hence speed is a scalar quantity.

To illustrate, if we say that our airplane is flying 200 km/hr due north, we are stating its velocity. But if we say that it is moving 200 km/hr relative to the ground, we are stating its speed.

All right. A boy runs diagonally across the junction of 5th Ave. and 34th St. at 20 ft per sec.

(23)

A This statement gives the boy's velocity.

B This statement gives the boy's speed.

YOUR ANSWER --- A

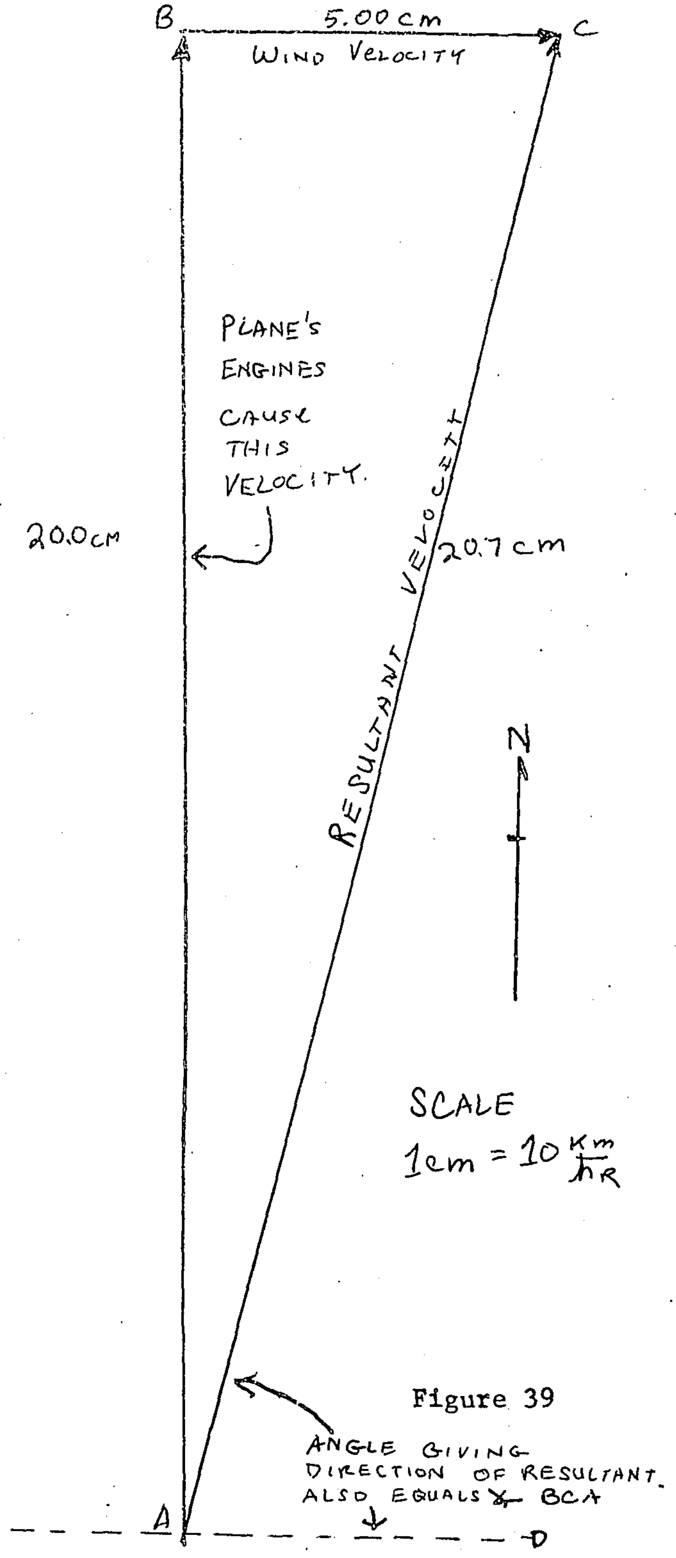


Figure 39

You could not have obtained this answer if your construction were correct and your measurement accurate.

Look over the correct diagram at the left. Compare particularly the lengths and directions of the line segments you drew as AB and BC with ours. If you did these incorrectly, repeat the diagram. You will see that your resultant vector line segment, AC, will be the same length as ours.

When your diagram is a close duplicate of Figure 39, measure the angle indicated very precisely to the nearest degree. Note that you might also measure angle BCA since this is equal to the exterior angle CAD.

Please return to page 18 and select another answer.

YOUR ANSWER --- C

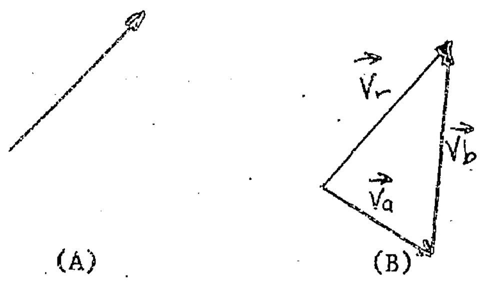


Figure 41

You are correct. From our definitions, the resultant of a pair of components is the vector sum of those components. We're just working backwards in this example. The resultant $\vec{v}_r$ could definitely have been obtained from the components $\vec{v}_a$ and $\vec{v}_b$ since $\vec{v}_a$ and $\vec{v}_b$ have been added by correct vector procedure.

Now, the next question is: "Are $\vec{v}_a$ and $\vec{v}_b$ the <u>only possible</u> pair of components that could, when added, produce a resultant $\vec{v}_r$?"

Study Figure 42. <u>All of the $\vec{v}_r$ vectors in this diagram are identical.</u>

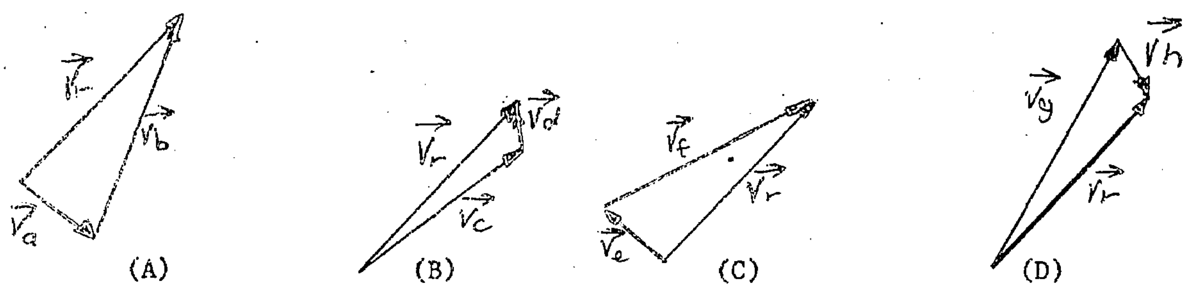


Figure 42

We have seen that $\vec{v}_a$ and $\vec{v}_b$ were correctly added to yield the resultant $\vec{v}_r$. (Figure 42A.) Now examne B, C, and D above with a view to determining whether or not we could properly call $\vec{v}_c$ and $\vec{v}_d$ another pair of possible components, and similarly, $\vec{v}_e$ and $\vec{v}_f$, and similarly $\vec{v}_g$ and $\vec{v}_h$.

The question is, will these pairs $(\vec{v}_c + \vec{v}_d)$, $(\vec{v}_e + \vec{v}_f)$, and $(\vec{v}_g + \vec{v}_h)$ serve equally well as possible components of the same $\vec{v}_r$?

(28)

A No.

B Yes.

YOUR ANSWER --- C

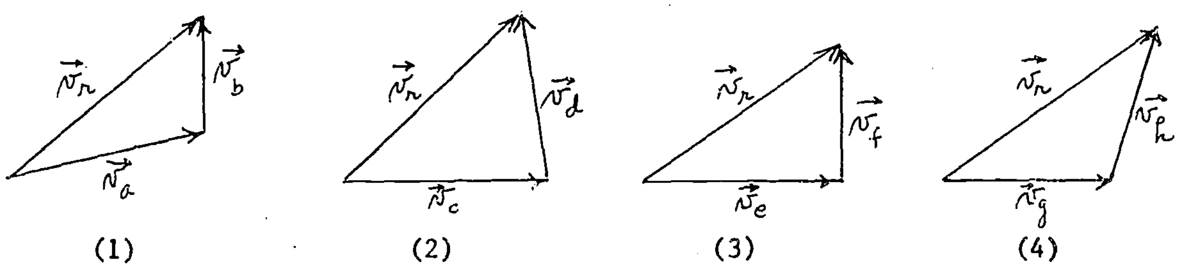


Figure 45

Incorrect. Look at (1) carefully. You will note that v_b is vertical but that v_a is <u>not</u> horizontal. <u>This makes both $\vec{v}_a$ and $\vec{v}_b$ incorrect components.</u>

You might think that $\vec{v}_b$ is the correct vertical component merely because it is a vertical line segment. This is not true because, when $\vec{v}_a$ is dropped to the horizontal position, where it should be according to the terms of the problem, then $\vec{v}_b$ increases in magnitude.

Thus, (1) does not represent $\vec{v}_r$ resolved into its vertical and horizontal components.

Please return to page 117 and select another answer.

YOUR ANSWER --- B

This answer is not wrong, yet we are unhappy that you didn't apply the rules. Sure, if you walk 11 ft one way and then 4 ft in the opposite direction, your displacement would be 7 ft. The trouble is that describing displacement as 11 ft - 4 ft may lead to sloppy habits. It's better to make a habit of following rules, so you'll know them when you need them.

When calculating the displacement resulting from two or more motions along a straight line, always assign a (+) sign to one direction and a (-) sign to the other direction. It doesn't matter which one you choose for the (+) and which for the (-). Now, if both trips are in the same direction, both would be assigned (+) signs and the displacement would be the numerical sum of the two. If they are in opposite directions, the algebraic sum would then give the numerical difference between the two if the correct sign convention is used. So, although your answer is not too bad, it is not the best way to do it.

Please return to page 47 and select the correct answer.

YOUR ANSWER --- A

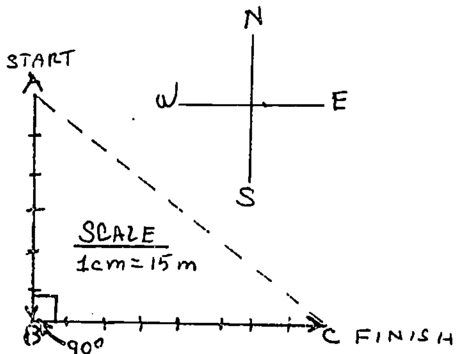


Figure 12

You're looking at the wrong vector. His total displacement is measured from A to C along the line segment AC. Count the number of centimeters in this line segment. What do you get? Now multiply this figure by the scale value (15 m/cm) to obtain the total displacement of the boy after the trip is complete.

Please return to page 103 and select another answer.

YOUR ANSWER --- C

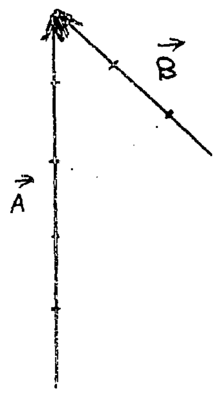


Figure 32

The diagram in Figure 32 is not right. $\vec{B}$ has been shifted, but its direction has not been reversed.

In the original problem diagram, $\vec{B}$ was directed toward the northwest. In Figure 32, where $\vec{B}$ is supposed to be reversed in direction, this vector is still pointing in a general northwesterly direction.

Please return to page 127 and select another answer.

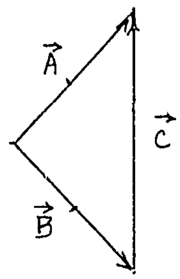
YOUR ANSWER --- A

You are not using the definition of displacement properly. A displacement is a vector quantity that shows how far and in what direction something has moved from its original position; that is, it indicates the present location of the object with respect to its starting position, its new "place." The distance is measured along a straight line from the starting position to the finishing position.

The answer you chose gives the length of the trip, not the displacement and also describes the direction moved on each leg of the trip. There is nothing wrong with such a description, but it is not the same as the displacement.

Please return to page 5 and select another answer.

CORRECT ANSWER: If diagram (2) were supposed to show a vector addition, the vectors would be connected head-to-tail, not tail-to-tail. The tail-to-tail connection, as you have seen, may be used as a short-cut in vector subtraction, but it is <u>never</u> used in addition.



(2)

Figure 38

So, please return to page 56.

CORRECT ANSWER: The 2-cm vector line segment represents 6 km of displacement.

Going a step farther, you can see that a vector line segment 3 cm long according to this scale must represent 9 km since each centimeter stands for 3 km. That is, the scale says, "There are 3 km for each cm of line length." We can write this in our familiar unit notation this way:

Scale: 3 km/cm or 3 km per cm

Using the figures above, we can then write, for a line segment of 3 cm length:

3 km/cm x 3 cm = 9 km

The same process gives us the answer to this question: "How many kilometers does a vector line segment 4 cm long represent according to the scale 1 cm = 3 km?" Thus:

3 km/cm x 4 cm = 12 km

On the same basis, a vector line segment of 20 cm would then represent a displacement of how many kilometers? The scale is still 1 cm = 3 km.

Please turn to page 73 to check your answer.

YOUR ANSWER --- D

This answer is incorrect. Refer to diagrams (1) and (4) reproduced here. In this manipulation of the vectors, $\vec{B}$ has been shifted parallel to itself without changing its direction so that its tail joins the head of $\vec{A}$. You should remember that this is exactly the procedure we use when two vectors are to be added to each other.

So, if vector $\vec{C}$ in diagram (4) does not represent the difference between $\vec{A}$ and $\vec{B}$, just what does it represent?

(1) (4)

Figure 38

Please turn to page 91 to check your answer.

YOUR ANSWER --- C

You are correct. Displacement is a vector quantity showing how far and in what direction something has moved from its original position. The distance corresponding to the magnitude of the displacement is measured along a straight line from the starting position to the finishing position.

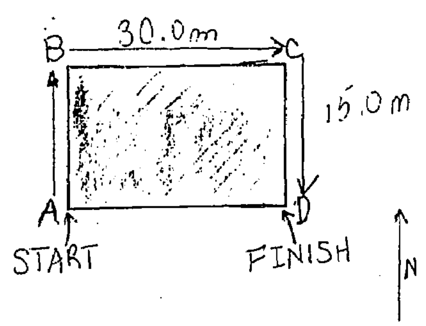


Figure 4

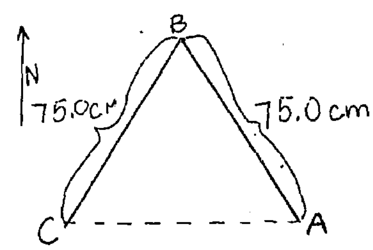


Figure 5

Thus, distance AD is the magnitude of the displacement and "to the east" describes the direction of the displacement.

Refer to Figure 5, showing an equilateral triangle. An ant, starting from point A, crawls along the straight line to B, and thence to C. Which one of the following statements describes this motion?

(2)

A The ant's trip length was 150 cm and his displacement was 75.0 cm east.

B The ant's trip length was 150 cm and his displacement was 75.0 cm west.

C The ant's trip length was 150 cm and his displacement was 75.0 cm.

YOUR ANSWER --- A

No, it doesn't. To be velocity, a statement of displacement per unit time, or rate of displacement, must clearly indicate the direction of motion. To one who is not familiar with the junction of 5th Ave. and 34th St., the specification "diagonally" means absolutely nothing. Furthermore, such a junction must have at least 2 alternative diagonal paths. Thus, the statement does not clearly delineate the direction of the boy's path and, therefore, it does not give the boy's velocity.

Please return to page 33 and select another answer.

YOUR ANSWER --- A

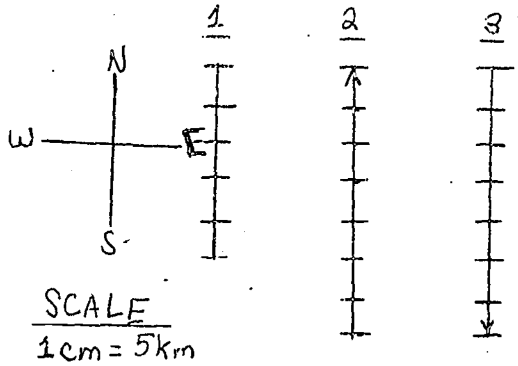


Figure 11

You are correct. The direction of the displacement is toward the south. The vector is 7 cm long and, with each centimeter representing 5 km, the total magnitude is: 5 km/cm x 7 cm = 35 km. Thus both requirements are met.

A few simple rules for working with vectors can now be derived from the foregoing examples.

NOTEBOOK ENTRY

2. Rules for Working with Vectors

(a) The displacement resulting from two trips in the same direction along a straight line is the algebraic sum of the two trip lengths. Arbitrary signs are given to each trip. (+) in both cases since they are in the same direction.

(b) The displacement resulting from two trips in opposite directions along a straight line is the algebraic sum of the two trip lengths. Arbitrary signs are given to each trip. (+) for one and (-) for the other since they are in opposite directions.

(c) The direction of a vector may be specified by stating:
   (1) compass direction
   (2) angle, measured from the X-axis in the 1st quadrant in a counterclockwise direction.

(d) The magnitude of a vector may be specified by means of a scale in which 1 cm = some convenient part of the total magnitude. The scale should be chosen so that the finished vector drawing is as large as possible without running off the written page.

Please go on to page 47.

If you were to place your back against the front wall of your room and walk 11 ft toward the rear of the room, then turn around and walk 4 ft toward the front of the room again, your displacement would be best shown by which one of the following?

(6)

A (+11 ft) + (-4 ft)

B 11 ft - 4 ft

YOUR ANSWER --- B

This was a guess. Rule 2(c) reads: The direction of a vector may be specified by stating:

(1) compass direction.
(2) angle, measured from the X-axis in the 1st quadrant, in a counterclockwise direction.

Please return to page 79 and select another answer.

YOUR ANSWER --- A

This is not a good choice.

The scale governs the size of the vector diagram. In this case the northerly velocity is 200 km/hr; hence you would need a line segment 100 cm long to represent the airplane's velocity. Since 100 cm = 1 meter and a meter is over a yard long, you have chosen a quite unsuitable scale to solve the problem on paper. If you were working on a blackboard, it might be very good.

Please return to page 89 and choose a scale that will give you a large diagram, but not so large as to require special paper.

YOUR ANSWER --- B

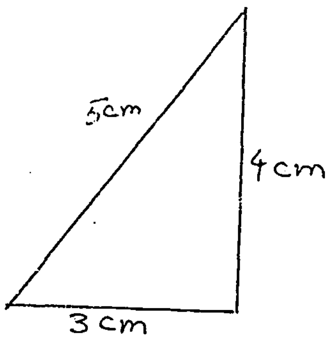


Figure 15

You have forgotten how to deal with 3:4:5 triangles. Well, let's run through the features of this triangle with the help of Figure 15. If a triangle has legs of 3 cm and 4 cm, respectively, then, using the Pythagorean theorem, we can quickly show that the hypotenuse must be 5 cm in length. The theorem tells us that the hypotenuse $= \sqrt{9 + 16} = 5$ cm.

Now imagine that we double the length of each of the legs to 6 cm and 8 cm, respectively. Using the Pythagorean theorem again, we find that the hypotenuse is now 10 cm long. Originally the sides were in the ratio 3 is to 4 is to 5. In the larger triangle the ratio is 6 is to 8 is to 10, but if we divide each of these figures by 2, we come back to 3:4:5 again. If we triple the length of the legs they become 9 cm and 12 cm, respectively. To find the hypotenuse, you need merely do this: say to yourself that if 9 and 12 are both divided by 3, they form the ratio 3:4. Therefore, the hypotenuse must have a value such that, when it is also divided by 3, the answer will be 5 (that is, to complete the 3:4:5 relationship).

Now assume that the legs are quadrupled to 12 cm and 16 cm respectively. The hypotenuse must therefore have a value such that when it is divided by 4, the answer will be ___?

Please turn to page 82.

YOUR ANSWER ---- C

Incorrect! Item 3(b) reads: When the sign of a vector is changed from + to -, the direction of the vector must be reversed.

Please return to page 17 and select another answer.

YOUR ANSWER --- C

This line is incorrect.

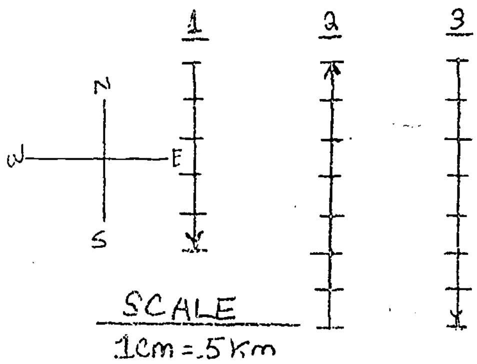


Figure 11

A southerly displacement of 35 km is to be represented according to the scale: 1 cm = 5 km. Now look at 1. The vector is, in fact, directed toward the south. But if each cm stands for 5 km and you have 5 cm altogether making up the line segment, then this represents a total vector magnitude of 5 km/cm x 5 cm = 25 km. But the specification was that you illustrate a displacement of 35 km to the south, not 25 km. Hence, your selection was in error.

Please return to page 98 and select another answer.

YOUR ANSWER --- C

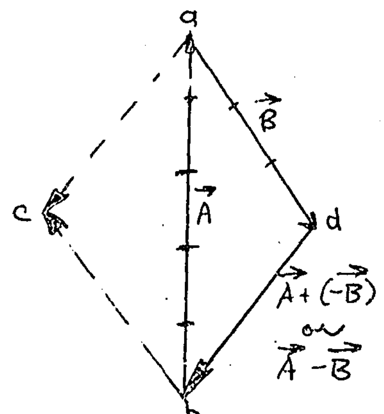


Figure 33

You are correct. In shifting $\vec{B}$ up to its new position, it was moved parallel to itself and was not changed in length; hence ad is equal and parallel to cb by construction. If one pair of opposite sides of a quadrilateral polygon are equal and parallel then the figure is a parallelogram. Hence, side db is equal and parallel to side ac, proving our point that the short-cut method of obtaining the difference vector by drawing in the line segment ac gives identical results with the procedure of reversing sign, shifting the vector to a head-to-tail connection, and then adding vectorially.

So--whenever you run into a vector subtraction, if the short-cut method is available to you, you may use it confidently. It always works.

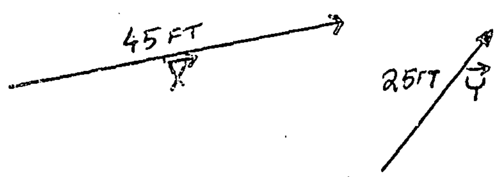


Figure 35

Let's consider one more example of vector subtraction. In Figure 35 are two vectors, $\vec{X}$ and $\vec{Y}$. $\vec{Y}$ is to be subtracted from $\vec{X}$; that is, you are to determine the result of $\vec{X} - \vec{Y}$. Using either method outlined previously, set up a suitable scale and determine the magnitude of the vector difference. (Note: if $\vec{X}$ is extended to meet Y, the angle between the vectors would be exactly 45°.)

Please turn to page 55 and compare your result with ours.

CORRECT ANSWER:

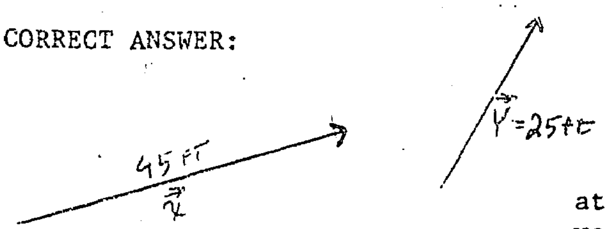


Figure 35

Figure 35 has been reproduced at the left to display the original vectors. $\vec{X}$ is 45 ft and $\vec{Y}$ is 25 ft. The directions of the vectors and their relationship to each other are shown by the line segments and arrows. You were to subtract $\vec{Y}$ from $\vec{X}$.

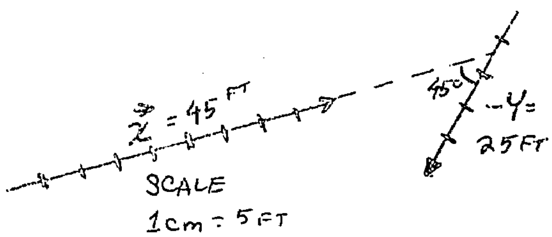


Figure 36

Figure 36 shows both vectors drawn to the scale: 1 cm = 5 ft. Thus, $\vec{X}$ is 9 cm long and $\vec{Y}$ is 5 cm long when drawn to scale. In addition, $\vec{Y}$ has had its direction reversed to convert it to $-\vec{Y}$. Now to obtain $\vec{X} - \vec{Y}$, we will perform the vector operation $\vec{X} + (-\vec{Y})$.

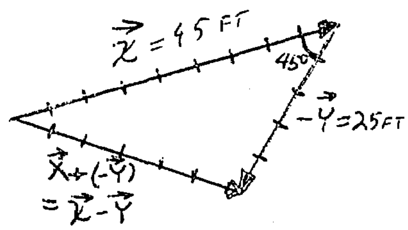


Figure 37

In Figure 37, $-\vec{Y}$ has been shifted parallel to itself so that its tail joins the head of $\vec{X}$. The line segment that completes the triangle is then $\vec{X} + (-\vec{Y})$ or $\vec{X} - \vec{Y}$. This vector turns out to be 6.5 cm long, which represents a length of 32.5 ft. We can round it off to 33 ft since the precision of the original data was to 2 significant figures.

Please go on to the next page.

Study the sequence of diagrams in Figure 38. In diagram (1) are two vector quantities $\vec{A}$ and $\vec{B}$. Diagrams (2), (3), and (4) show various operations with $\vec{A}$ and $\vec{B}$. Establish firmly in your mind the relationship of each of the diagrams to the original vectors. Then answer the questions below.

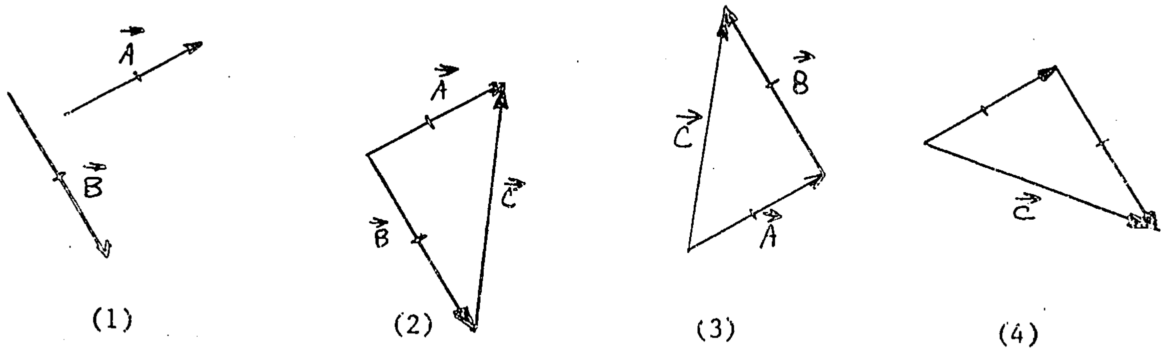


Figure 38

Of the statements below, which is the only <u>true</u> one?

(21)

A Diagram (2) illustrates proper addition of $\vec{A}$ and $\vec{B}$.

B Diagram (3) illustrates proper addition of $\vec{A}$ and $\vec{B}$.

C Diagrams (2) and (3) illustrate proper subtraction of $\vec{B}$ from $\vec{A}$.

D Diagram (4) illustrates proper subtraction of $\vec{A}$ from $\vec{B}$.

YOUR ANSWER --- C

You are correct. At this stage of your training, it is advisable to show vector subtractions as vector additions with one of the signs changed, just as you did it in the answer above.

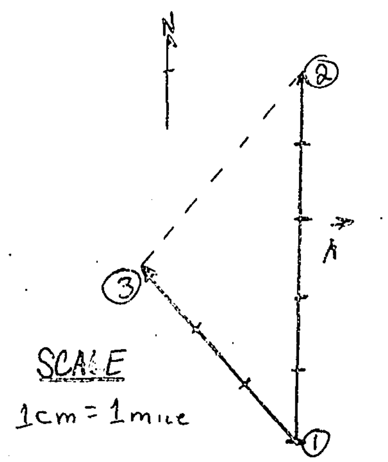


Figure 29

You should now be ready to tackle the subtraction of vectors that are not collinear; that is, vectors that do not lie along the same straight line.

Referring to Figure 29, we see that a car has again made a 5 mi trip starting from point (1), going due north, at some given time, as shown by $\vec{A}$. Later, the car takes off from the same starting point and travels at some unspecified angle in a general north-westerly direction for a distance of 3 mi. We are interested in answering the question, "What is the difference in position of point (3) in both magnitude and direction relative to point (2)?"

Now, it is almost obvious that if you drew a vector line segment from (2) to (3), placing the arrow at (3), this vector would accurately show the magnitude and direction of the vector difference between A and B. It's common sense! Isn't point (3) located to the southwest of (2) by a distance equal to the line from (2) to (3)? Of course it is! But, in getting the vector difference this way, we have flagrantly violated one of the procedural rules for working with vectors. Which rule did we violate? (See notebook entries 2(e) and 3(c).)

(18)

A We violated the rule that describes how the scale should be used.

B We violated the rule that describes how vectors should be connected for addition or subtraction.

C We violated the rule that describes how to determine the direction of the final vector.

YOUR ANSWER --- A

This is incorrect.

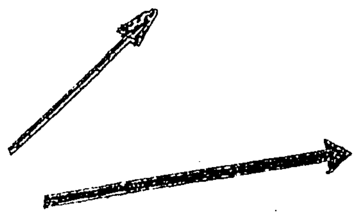


Figure 24

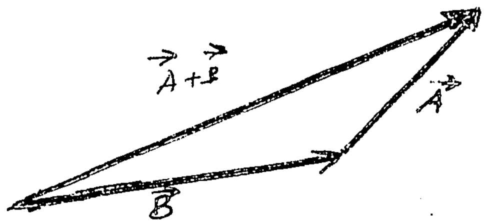


Figure 25

Be careful of falling for optical illusions. If necessary, use your ruler (scale) to check the magnitudes of the vectors. The magnitude of $\vec{A}$ in Figure 25 is precisely the same as the magnitude of $\vec{A}$ in Figure 24. This is also true of $\vec{B}$.

Thus, if Figure 25 contains an error, it does not involve a change of magnitude.

Please return to page 15 and select another answer.

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YOUR ANSWER --- B

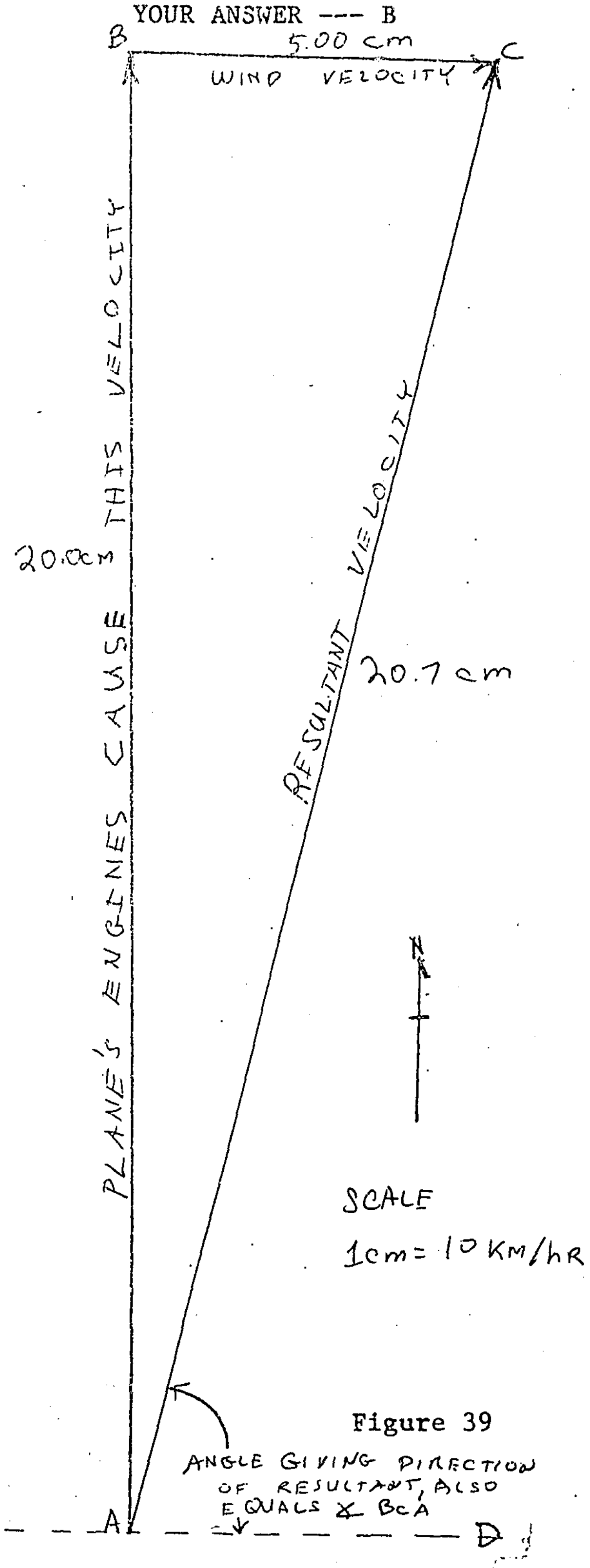


Figure 39

You could not have obtained this answer if your construction were correct and your measurement accurate.

Look over the correct diagram at the left. Compare particularly the lengths and directions of the line segments you drew as AB and BC with ours. If you did these incorrectly, repeat the diagram. You will see that your resultant vector line segment, AC, will be the same length as ours.

When your diagram is a close duplicate of Figure 39, measure the angle indicated very precisely to the nearest degree. Note that you might also measure angle BCA since this is equal to the exterior angle CAD.

Please return to page 18 and select another answer.

YOUR ANSWER --- A

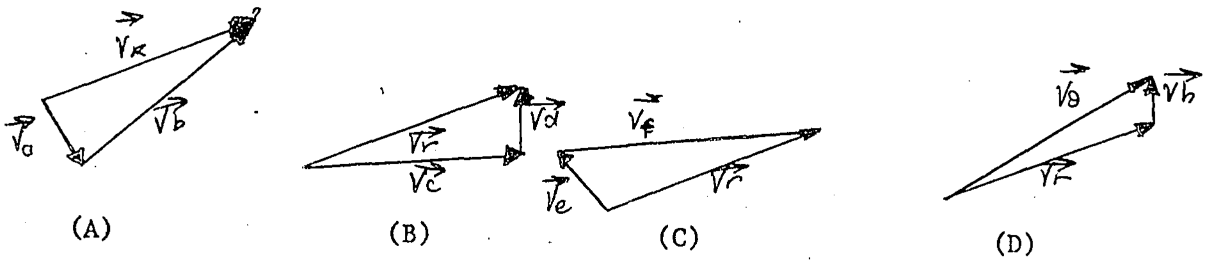


Figure 42

You did not study the diagrams with sufficient care. Here are the pairs:

$\vec{v}_a$ and $\vec{v}_b$
$\vec{v}_c$ and $\vec{v}_d$
$\vec{v}_e$ and $\vec{v}_f$
$\vec{v}_g$ and $\vec{v}_h$

The elements of each pair have been added by proper vector procedure to produce a resultant $\vec{v}_r$.

The fact that $\vec{v}_r$ is the same resultant in each case shows that a given velocity may be a resultant of <u>more than one pair of component velocities</u>.

Please return to page 35 and select the correct answer.

YOUR ANSWER --- A

You didn't follow the explanation. The entire line measures 8 cm but represents 96 km drawn to scale. We want to know the displacement that each centimeter of the line segment represents. This is comparable to the question:

"If 75 cents represents the cost of 25 apples, how much does each apple cost?" In this question, of course, you simply divide the cost of the group (75 cents) by the number in the group (25 apples) to obtain the cost of a single apple. Thus each apple costs 75¢/25 or 3¢.

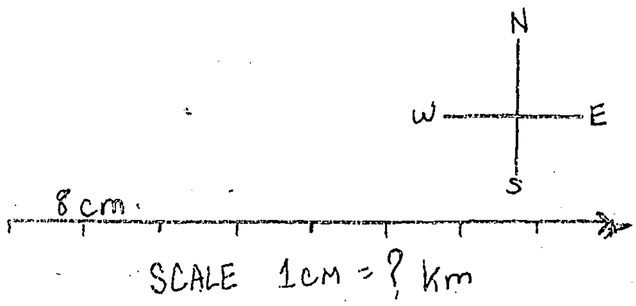


Figure 8

Similarly, in the vector problem, if the 8-cm line segment represents 96 km, then one centimeter must represent how many kilometers?

Please return to page 102 and select another answer.

YOUR ANSWER --- A

You are correct. The ratio of the legs is 20/48 or 5/12; hence the hypotenuse must complete the ratio 5:12:13. Since 20 is 4 x 5, and 48 is 4 x 12, then the hypotenuse must be 4 x 13 = 52 ft.

The Pythagorean theorem may be used to add vectors <u>only</u> when the two original displacements are at right angles or 90° to each other. The graphical method of scale drawings, however, works well for <u>any</u> angle. It must be understood at this point that the graphical method seldom can be relied upon to give results as precise as those obtained by the mathematical approach. Nonetheless, the graphical solution is a good one because it has no angle limitations and because it yields good approximations quickly.

We'll try a problem based upon displacement at some angle other than 90° this time.

A car travels 4.0 miles due east, then changes direction and proceeds a distance of 3.0 miles <u>due southeast</u>. At the end of its trip, what is its displacement with respect to its starting point? Use a scale: 1 cm = 0.5 mi. In this case, as before, your scale drawing should give a numerical answer for the magnitude of the displacement and show, by the direction of the resultant vector, what the direction of the displacement is. Draw this diagram on a clean sheet of paper.

When you are finished, please turn to page 65 to compare your work with ours.

CORRECT SOLUTION: Refer to Figure 17 below.

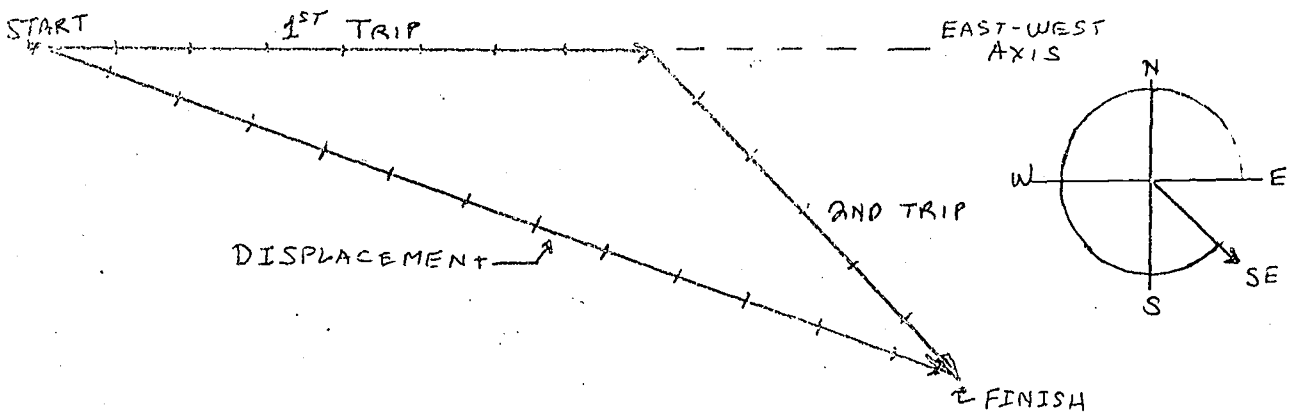


Figure 17

Inspect the correct solution diagram. The first trip consists of a 4.0 mi journey due east. We see that the vector line segment representing this trip points to the right (east) and is drawn 8.0 cm long, according to the scale: 1 cm = 0.5 mi. In the second trip, the car moves due southeast. This vector line segment makes an angle of 315° with the east-west axis according to the convention that governs angle measurement. The second trip segment is drawn 6 cm in length since it represents 3 mi.

When this drawing is very carefully made, we find that the total displacement vector is 13 cm long (to 2 significant figures as required by the precision of the data). Therefore, the total displacement of the car may be found using this figure and the scale.

What is the total displacement magnitude to 2 significant figures?

(13)

A The total displacement magnitude of the car is 5.0 mi.

B The total displacement magnitude of the car is 6.5 mi.

CORRECT ANSWER: You inverted A, thereby changing the direction of the original vector.

The diagrams of the original vectors constitute <u>data</u>. Data may never be changed for any reason whatever. If the original vector is directed toward the north, as $\vec{A}$ is in this illustration, then the final diagram must also show it directed toward the north.

Now go back to the original set of diagrams and look for these 3 qualities in the correct one: (1) one of the vectors has been moved parallel to itself; (2) vector directions or magnitudes have not been altered; (3) the resulting diagram shows vectors joined head-to-tail.

Please return to page 120 and select another answer.

CORRECT ANSWER: Vector $\vec{C}$ is the difference between $\vec{A}$ and $\vec{B}$ and is the answer to the operation $\vec{A} + (-\vec{B})$ or $\vec{A} - \vec{B}$.

This is the proper way to obtain a vector difference. The subtrahend (in this case $\vec{B}$) is changed in sign, which means that its direction is reversed. It is then shifted parallel to itself to achieve a head-to-tail connection with the unaltered vector. The line segment that completes the newly constructed triangle is the difference. Isn't that exactly the process used in obtaining diagram (3) from diagram (1)? It certainly is.

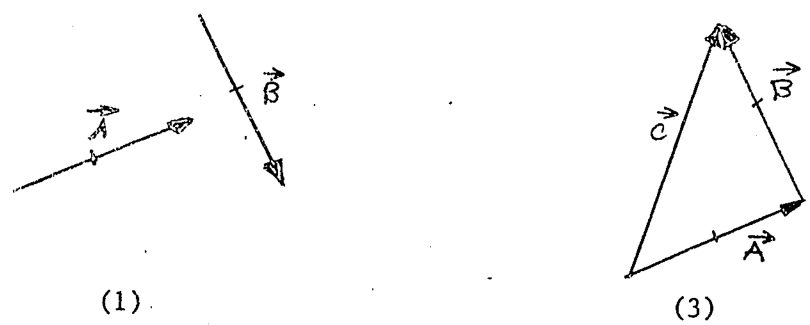


Figure 38

Please return to page 56 and select another answer.

YOUR ANSWER --- B

Right. The resulting vector, AC, represents his total displacement. It measures 10 cm, hence 15 m/cm x 10 cm = 150 m. This method of finding resulting displacement is known as the graphical method because it involves setting up a scale drawing from which the solution is found by measurement. The direction of the resulting vector AC is the direction of the total displacement. Unless you are asked specifically to find the angle of this displacement, the picture itself is enough to show the direction of the resulting vector.

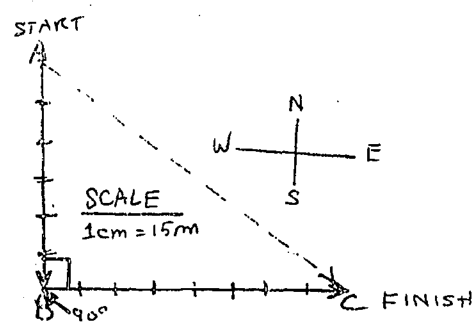


Figure 12

The graphical solution shows that a trip directly from A to C gives the same result as the two trips, AB and BC, separately. To show this we can write:

trip AB + trip BC = trip AC

The (+) sign in this case, however, doesn't have the same meaning as in ordinary arithmetic or algebra. That is, trip AB = 90 m and trip BC = 120 m, but certainly 90 m + 120 m does not yield 150 m by algebraic addition. So, we call this vector addition.

Please go on to page 69.

To show that a quantity is a vector without constantly writing the word "vector," we use a universal convention. That is, instead of writing "vector AB + vector BC = vector AC," we substitute this:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

Thus, any line segment carrying an arrow above its designation is a vector quantity. We will use full arrowheads as shown above, although some authors use half-arrowheads like this $\rightharpoonup$ to represent the same thing.

Here is your next job. Prepare a sheet of clean paper on which you will solve the following problem:

A car travels due east a distance of 45.0 km, then due north a distance of 105 km. Using this scale, 1 cm = 15 km, construct the vector diagram which will enable you to determine the resulting displacement of the car at the end of both trips. Use a compass of protractor to insure the proper 90^{o} relationship between the trips. Your pencil should be very sharp, your measurements very precise, and the lines relatively light. A good vector diagram has fine lines, not broad, sloppy ones. You may measure to the nearest millimeter, which will give you measurements to 2 significant figures.

O.K., draw your picture; then turn to page 104 to see what our drawing looks like.

YOUR ANSWER --- B

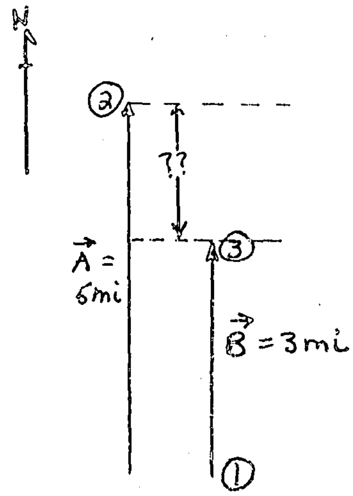


Figure 27

You're using the wrong reference point. We asked for the difference in position of point (3) relative to point (2). You have given us the difference in position of point (3) relative to point (1). Sure, point (3) is 3 mi north of point (1), but how many miles and in what direction is point (3) relative to point (2)? Which direction is point (3) from point (2)?

Please return to page 126; choose the right answer by noting the reference point requested.

YOUR ANSWER --- B

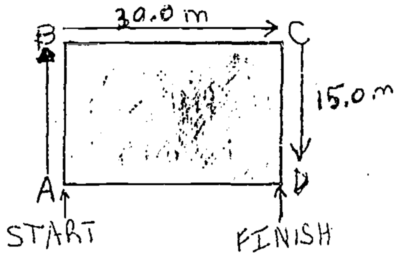


Figure 4

That's incorrect. Displacement is a vector quantity that shows how far and in what direction something has moved from its original position. The distance corresponding to the magnitude of the displacement is measured along a straight line from the starting position to the finishing position.

Now look at the diagram. Point D, the finishing position, is truly 30.0 m from point A, the starting position. Hence, the <u>magnitude</u> of the displacement is definitely 30.0 m. But giving the directions traveled in each leg of the journey does <u>not</u> express the direction of the displacement in accordance with the description in the first paragraph above. You have the displacement magnitude right, but the directions mentioned indicate a perimeter which might be 30.0 m on each of three sides.

Please return to page 5 and select another answer.

YOUR ANSWER --- C

There is no rule that describes this. Actually, if all the rules are carefully followed, the direction of the final vector is shown automatically. It may be that the final vector, the line segment joining (2) to (3), has the wrong direction, but this is not the result of misuse of a special "direction" rule.

No, you missed the point. The rule we have violated is much more definite than your choice of answer indicates. Think a bit more; you will recognize the violated rule.

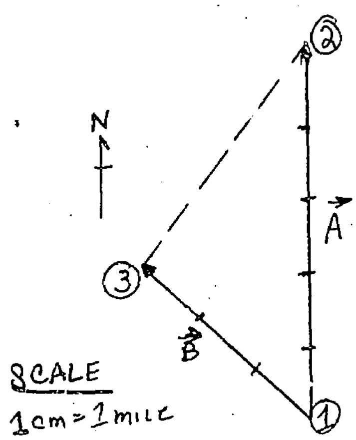


Figure 29

Please return to page 57 and select another answer.

CORRECT ANSWER: A vector line segment 20 cm long would represent 60 km on the basis of a scale of 1 cm = 3 km.

Thus, to find the total magnitude of a vector you simply multiply the scale numeral by the total length of the line segment.

For example, let's assume the scale to be 1 cm = 14.3 meters. Then a vector line segment 6 cm long would indicate a total vector magnitude of:

$$14.3 \text{ m/cm} \times 6 \text{ cm} = 85.8 \text{ m}$$

Going back to our original question, we asked how many kilometers are represented by each centimeter of a vector line segment 8 cm long if the total vector magnitude is 96 km. To find the answer, we use a process that is the reverse of the one we have just discussed. That is:

$$\frac{96 \text{ km}}{8 \text{ cm}} = \text{??? kilometers } \underline{\text{for}}\ \underline{\text{each}} \text{ centimeter of line segment length.}$$

Please return to page 102 and select the correct answer.

YOUR ANSWER --- A

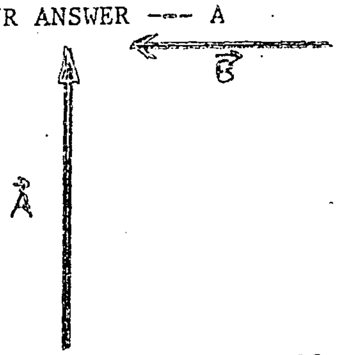


Figure 20

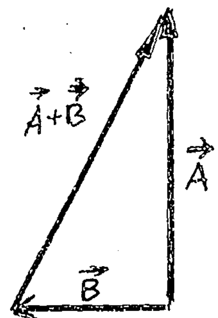


Figure 21

It shows the proper combination of the vectors in Figure 20 for the purpose of obtaining the vector sum.

The principle which describes the correct method of shifting vectors for the purpose of obtaining their sum states that either vector may be moved parallel to itself until the two are joined head-to-tail.

In your diagram choice, vector $\vec{B}$ has been moved parallel to itself in conformity with the first part of this principle. But in the final position $\vec{A}$ and $\vec{B}$ are joined <u>tail-to-tail</u>. This does not conform to the latter portion of the principle; hence the diagram does not give the right sum. The sum $\vec{A} + \vec{B}$ may have the right magnitude, but its direction is wrong.

Please return to page 120 and select another of the diagrams.

YOUR ANSWER --- B

This answer might seem correct at first glance, but a bit of thought will demonstrate its error. In Figure 6, we have the vector of our example. Although it is quite true that this vector makes an angle of 45° to the north-south axis, such a description does not <u>uniquely determine</u> this particular vector.

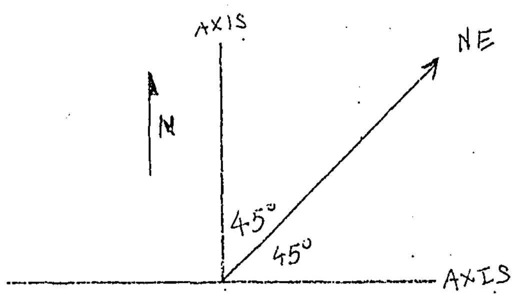


Figure 6

Refer now to Figure 7 below. You will se at once that the direction of this particular vector also makes an angle of 45° to the north-south axis: hence this description fits two entirely different directions. This, of course, is also true of a line described from the east-west axis, without further considerations. You should recall from algebra that angle measurements always start in the first quadrant along the X-axis. Only the first quadrant contains angles between 0° and 90°. Furthermore, all angles thereafter, regardless of the quadrant in which they appear, are measured from the X-axis in the first quadrant, in a counter-clockwise direction.

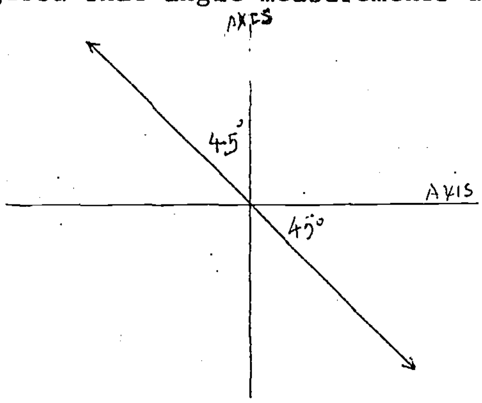


Figure 7

Please return to page 13 and select another answer.

YOUR ANSWER --- A

Not so. This answer does not agree with our explanations thus far. As long as a pair of components are correctly added vectorially, and as long as this addition yields the resultant in question, then these added vectors may be considered to be components of the particular resultant under discussion.

We have already shown 4 different pairs of vectors that may be considered components of a given resultant $\vec{v}_r$. Here are three more such combinations.

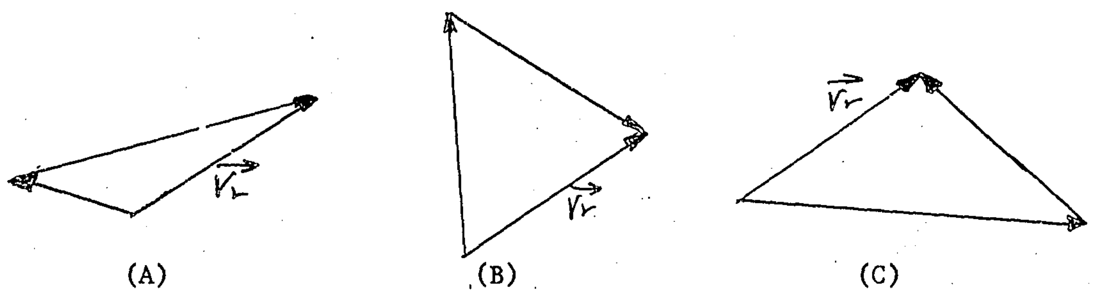


Figure 43

In the face of these simple examples, it is impossible to say that a given vector can be obtained from only one pair of possible components.

Please return to page 25 and select another answer.

YOUR ANSWER --- C

You've skipped an important characteristic of vectors. Although the trip length is a scalar with no direction specified, the displacement is a <u>vector</u> quantity and direction <u>must</u> be described.

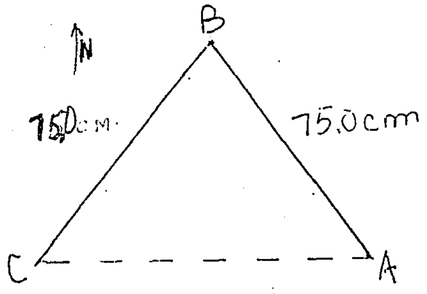


Figure 5

You will note that the displacement is given as 75.0 cm. This is the correct magnitude for the displacement vector, but it does not describe it fully because it omits a statement of the direction of displacement.

The direction should be specified from the point of view of an observer at the starting point, looking toward the finishing point. If you were the observer at A, in which direction would you have to cast your glance to spot the ant after he has arrived at his destination, point C? This is the direction of the displacement vector.

Please return to page 44 and select another answer.

YOUR ANSWER --- A

You are correct. The resultant displacement is represented by $\overrightarrow{KM}$, which is 7.6 cm long. Thus, 15 km/cm x 7.6 cm = 114 km.

It has probably occurred to you that the displacement resulting from two trips at 90^{o} to each other might be calculated by algebraic methods, making a scale drawing entirely unnecessary. This is quite true. You will recall one of the most fundamental theorems in geometry which says that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the arms. This is the Pythagorean theorem, of course.

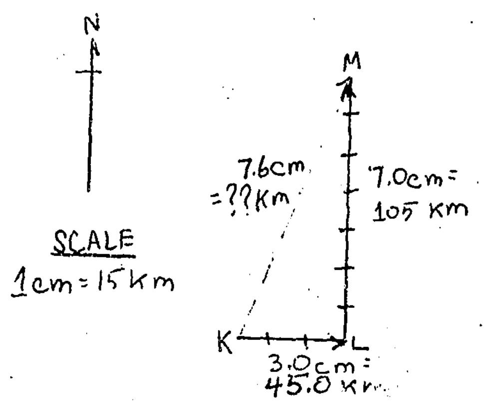


Figure 13

Let's use the Pythagorean theorem to check the last problem. Figure 13 is repeated here to remind you of the problem.

The hypotenuse of the right triangle KLM is the line segment KM; the arms are, respectively, KL and LM. Thus, from the Pythagorean theorem we have:

$$(KL)^2 + (LM)^2 = (KM)^2$$

We substitute the known quantities:

$$(45.0)^2 + (105)^2 = (KM)^2$$
$$2,025 + 11,025 = (KM)^2 = 13,050$$

and we get: $KM = \sqrt{13,050} = \underline{114}$

This checks nicely with the graphical solution. When done this way the solution is said to be mathematical rather than graphical.

Before continuing, please turn to page 142 in the blue appendix.

NOTEBOOK ENTRY

(topic 2)

(e) The displacement resulting from two trips at any angle to each other may be found by adding the representative vectors in graphical form. The vectors are connected tail-to-head; the remaining side of the triangle is the vector sum. The vectors must be drawn to scale.

(f) Two vectors at right angles to each other may be added mathematically by means of the Pythagorean theorem.

NOTEBOOK CHECK

Referring to Lesson 4, notebook entry item 2: Which of the four rules describes how the magnitude of a vector may be specified?

(10)

A Rule (d).

B Rule (c).

C Rule (b).

D Rule (a).

YOUR ANSWER --- A

Incorrect! Item 3(d) reads: The vector difference is then given by the magnitude and direction of the line segment joining the head of $\vec{B}$ to the tail of $\vec{A}$.

Please return to page 17 and select another answer.

YOUR ANSWER --- B

That's not right.

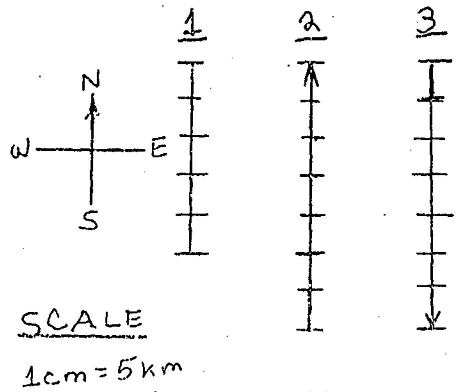


Figure 11

A southerly displacement of 35 km is to be represented according to the scale: 1 cm = 5 km. Now look at 2. The magnitude of the vector is obtained by counting the number of centimeters in the line segment and applying the scale. There are 7 cm in the line segment, and each centimeter has a magnitude value of 5 km. Hence, the total displacement is 5 km/cm x 7 cm = 35 km. Thus, the magnitude of the vector is correct.

However, the specified direction in the problem is south. The vector in 2 points toward the north. Thus, your error here was in choice of direction rather than magnitude.

Please return to page 98 and select another answer.

CORRECT ANSWER: In a triangle whose legs are 12 cm and 16 cm, respectively, the hypotenuse must have a value such that when divided by 4 (the quadrupling factor), the answer will be 5.

Let's check your understanding once more. Suppose the legs of a right triangle are 21 cm and 28 cm, respectively. To find out whether they are in the ratio 3:4, divide the smaller number by 3 and the larger by 4.

$$\frac{21}{3} = 7 \qquad\qquad \frac{28}{4} = 7$$

If this pair of quotients is the same, then the sides are in the ratio 3:4, and the hypotenuse must be such that you will obtain 5 when you divide it by the common multiplier for this group, namely, 7. Thus, if the legs are 21 cm and 28 cm, then the hypotenuse must be 35 cm, which is obtained by multiplying 5 from the ratio by 7, the common multiplier.

Please return to page 100 and work out the original problem.

YOUR ANSWER --- B

Incorrect. No further information about the magnitude of $\vec{v}_r$ is necessary because this is implicity given by the arbitrary length of the $\vec{v}_r$ vector in the diagram.

If $\vec{v}_r$ is drawn to a certain scale, then its components will be drawn to the same scale; hence all magnitude information is available in the form of scaled vector lengths.

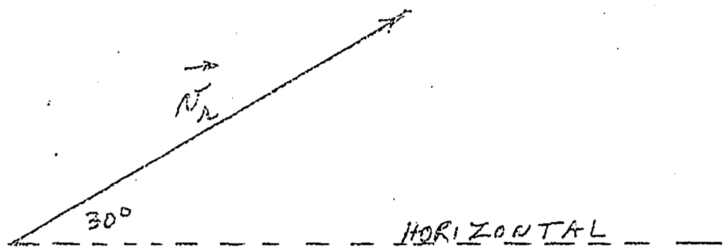


Figure 44

Please return to page 133 and select another answer.

YOUR ANSWER --- B

You are correct. Our original vector is the velocity of the swimmer, $\vec{v}_s$. We can find the velocity of the stream only by resolving $\vec{v}_s$ into two right angle components, one across the river and the other in the direction of the water movement. When this is set up, we see at once that a vector BC will tell us the direction and magnitude of $\vec{v}_w$.

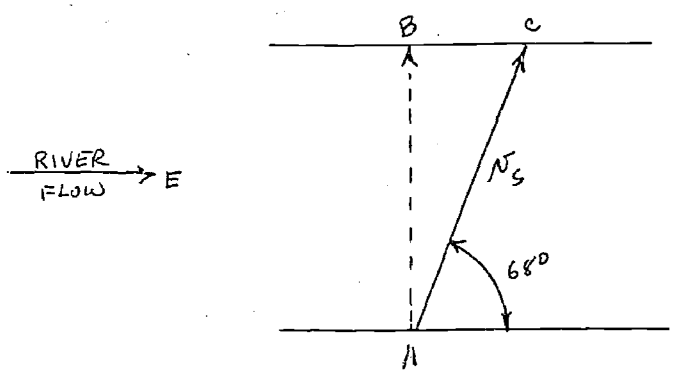


Figure 47

We'll assume then that you are making your drawing at this time. You should have drawn AC so that it makes an angle of 68.0^0 with the south bank and has a length scaled from $\vec{v}_s$ = 2.8 km/hr (See Figure 47). At what angle to AC will you now draw in the "straight-across" vector AB?

Please turn to page 95 to check your answer.

YOUR ANSWER --- A

You're not reading carefully. If the velocity of the swimmer is given in km/hr, how can you come out with an answer in terms of mi/hr without making a conversion from Metric to English? Besides, there is something else wrong with this answer. Your diagram doesn't seem to be accurate.

Please return to page 95 and select another answer.

YOUR ANSWER --- A

Wrong. You certainly didn't get this answer by applying the scale! We suspect you noticed that the resulting triangle has legs that are in the ratio of 3:4, and that you mistook it for a 3:4:5 triangle. Having made this mistake, you concluded that the longest leg of the triangle must be 5 mi.

The 3:4:5 ratio applies only to right triangles. This one clearly is not a right triangle; hence you cannot use the ratio.

Please return to page 65 and select the correct answer.

YOUR ANSWER --- D

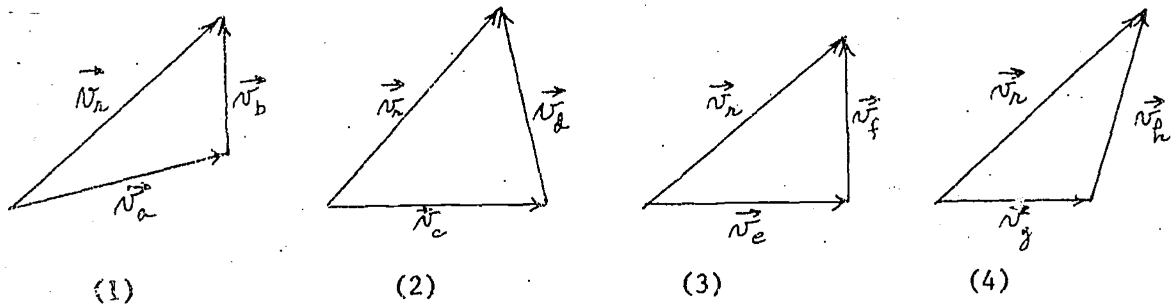


Figure 45

Incorrect. In (2), $\vec{v}_c$ is horizontal but $\vec{v}_d$ is not vertical. The conditions of the problem call for a right angle between $\vec{v}_c$ and $\vec{v}_d$. The angle, as you can see, is less than 90°. This makes both $\vec{v}_c$ and $\vec{v}_d$ wrong components.

You might think that $\vec{v}_c$ is the right horizontal component merely because it is a horizontal line segment. This is not true because, should $\vec{v}_d$ be rotated until it makes a right angle with $\vec{v}_c$, the latter would shrink in magnitude.

Thus, (2) does not represent $\vec{v}_r$ resolved into its vertical and horizontal components.

Please return to page 117 and select another answer.

YOUR ANSWER --- B

The operation could be shown this way since you have been directed to perform a subtraction. But, as we have pointed out, the actual technique of vector subtraction requires changing one of the vector signs so that we may perform an addition using the process we have developed so carefully.

At least for the present, you ought to write a vector subtraction according to the rules. That is, change the sign of the subtrahend and then add the vectors.

Please return to page 135 and select another answer.

YOUR ANSWER --- D

You are correct. Keep your notebook up-to-date at all times.

Velocity is a vector quantity. Vectors may be added by the graphical process we have learned, and velocity is no exception.

We'll consider an example, this time using the proper terms.

An airplane is flying due north at a speed of 200 km/hr. At the same time, a steady wind blowing to the east with a velocity of 50 km/hr acts upon the plane. What is the resultant velocity of the plane?

The independent motion of the plane due to its engines is given as a true velocity since both magnitude and direction are clearly stated. The same is true of the wind velocity. The question asks for the resultant or sum velocity which automatically informs you that you must give both the resultant speed and resultant direction of the plane.

The first step is the selection of a suitable scale. Which one of the following do you think represents the best choice, using a regular $8\frac{1}{2}$ x 11 inch piece of paper?

(25)

A 1 cm = 2 km/hr

B 1 cm = 10 km/hr

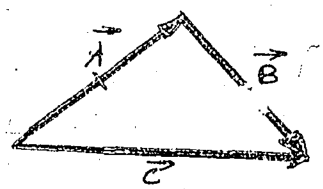
C 1 cm = 50 km/hr

D 5 cm = 5 km/hr

This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

CORRECT ANSWER: In diagram (4), vector $\vec{C}$ represents the sum of $\vec{A}$ and $\vec{B}$ in the operation $\vec{A} + \vec{B}$.

Since $\vec{B}$ has not been changed in sign (reversed in direction), then the process shown in (4) is addition. The sum of two vectors is obtained by connecting them head-to-tail as in diagram (4). The line segment that completes the newly formed triangle is the sum of the original vector quantities.



(4)

Figure 38

Please return to page 56 and select another answer.

CORRECT ANSWER: The resultant velocity of the boat is 47.5 km/hr, 22^{o} S of W.

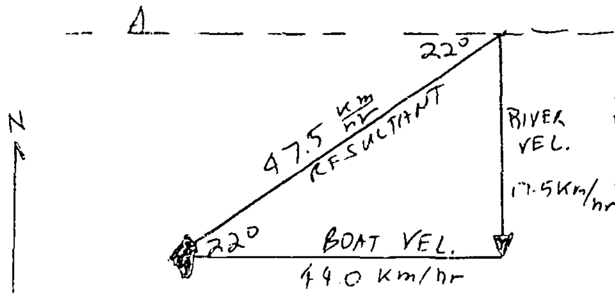
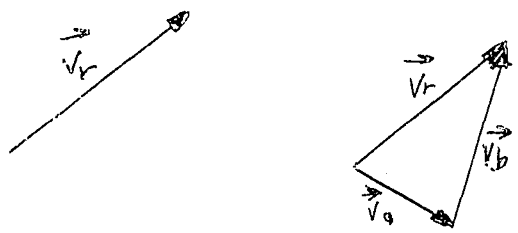


Figure 40

Refer to Figure 40. The solution should be quite clear from the diagram. Whether or not you selected a scale similiar to that in Figure 40 should make no difference whatever.

In the diagram of Figure 40, or in any other diagram of velocities that are related in a similar fashion, the two original vectors (in this case the boat velocity and the river velocity) are called the <u>components</u> or <u>component vectors</u>. The word "component" means "part of." In the example above, the boat and river velocities are each <u>part of</u> the resultant velocity; that is, they are <u>components</u> of the resultant velocity. The resultant velocity has been found by vector addition of the components.

At this point, we are going to reverse our approach. First note the velocity $\vec{v}_r$ in Figure 41A. Suppose you were told that this velocity was the resultant of two component velocities that had just been added by the triangle method which we have been using all along. Suppose that you were next shown the diagram of Figure 41B in which $\vec{v}_r$ is exactly the same as in Figure 41A. Would you say that $\vec{v}_a$ and $\vec{v}_b$ are components of $\vec{v}_r$?



(A) (B)

Figure 41

(27)

A I don't know.

B No.

C Yes.

CORRECT ANSWERS: (a) The skier travelled 4.33 km horizontally. That is, $s_h = 4.33$ km.
(b) The height of the top of the slope above ground level is 2.50 km. That is, $s_v = 2.50$ km.

So, you see, this problem is readily solved by the resolution-of-vectors approach. The total trip $\vec{s}$ may be considered to have two components: (1) the horizontal component $\vec{s}_h$, and (2) the vertical component $\vec{s}_v$.

Now we'd like you to try one on your own. Refer to Figure 47. A river flows due east as shown. A swimmer starting from point A on the south bank heads directly across the stream toward point B on the north bank. The river current, however, forces him to land on the north bank at point C. An observer on the south bank notes the swimmer's actual direction along AC and determines the angle between AC and the south bank to be 68.0°. The swimmer's speed is 2.8 km/hr. Using these data, find the velocity of the water, $\vec{v}_w$.

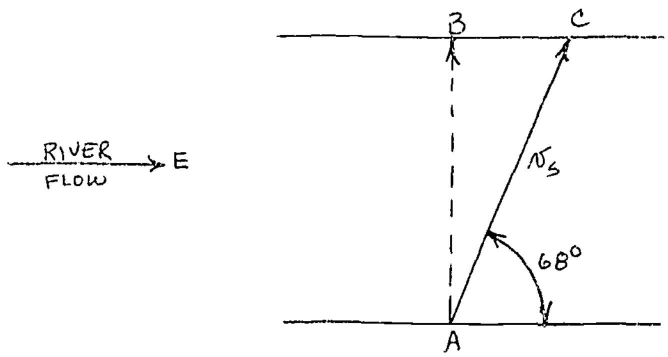


Figure 47

Which vector shows the direction and magnitude of $\vec{v}_w$?

(32)

A $\overrightarrow{AB}$.

B $\overrightarrow{BC}$.

C $\overrightarrow{AC}$.

YOUR ANSWER --- D

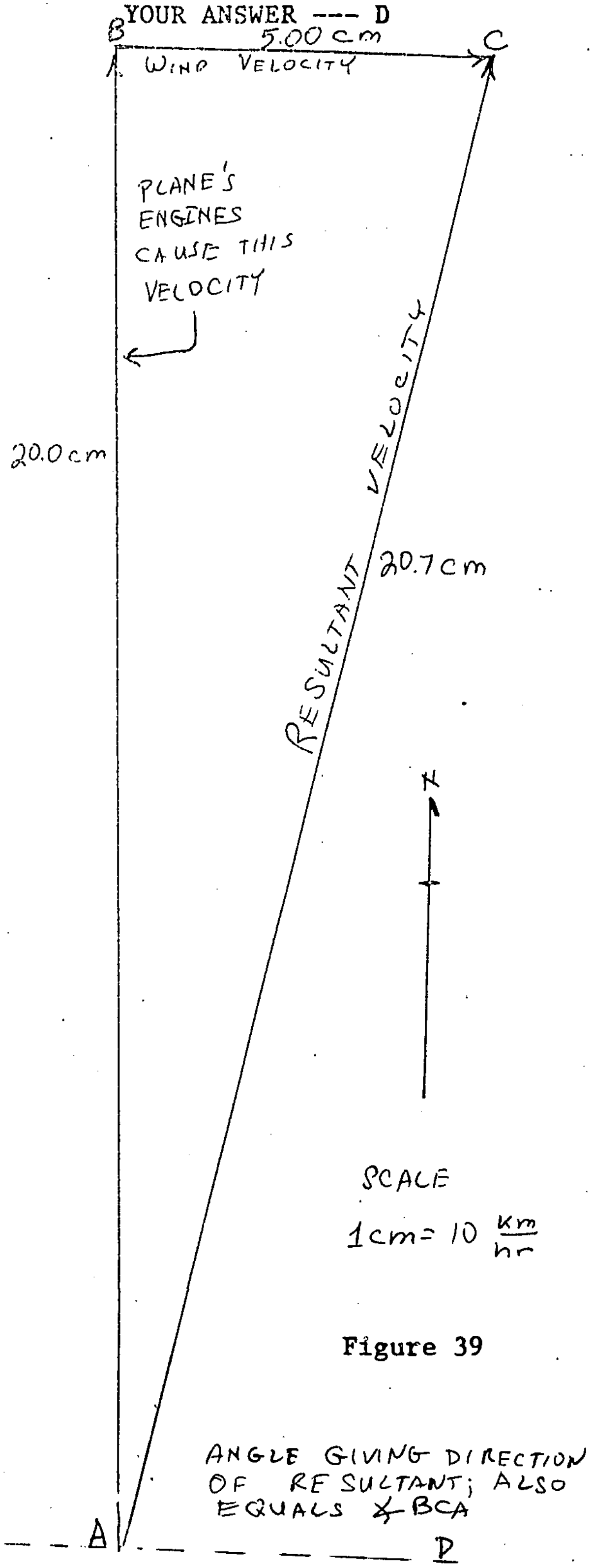


Figure 39

Well, it shouldn't have been. If your construction is correct and your measurements accurate, you should come out with an answer very close to one of those given.

Check the features of Figure 39. Note the lengths and directions of the two component vectors, AB and BC. Check their lengths on your drawing to be certain they match the scale. Make sure you construct angle CBA precisely so that it is truly a right angle. When you finally draw your resultant vector AC, measure it carefully. If the rest of your work is right, this line should be very nearly 20.7 cm in length. Then measure the angle indicated in Figure 39 to the nearest degree. Note that you might also measure angle BCA since this is equal to the exterior angle CAD.

Now, please return to page 18 and select the right answer.

CORRECT ANSWER: The angle between AC and AB should be 22.0° (that is, $90.0^{\circ} - 68.0^{\circ} = 22.0^{\circ}$).

Complete your drawing. Which one of the following gives the right answer to the problem, to 2 significant figures? (Note: due to a certain amount of inaccuracy in scaling and measurement, slight errors are bound to occur. Therefore, your answer may differ by a small amount from the correct one.)

(33)

A v_w = 0.98 mi/hr to the east.

B v_w = 1 km/hr to the east.

C v_w = 1.1 km/hr to the east.

D v_w = 0.98 km/hr to the west.

This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

YOUR ANSWER --- B

Not true. Altitude has no bearing on this particular problem. For example, imagine two planes in the air, one of them 2,000 ft higher than the other. Assume further that they are flying with constant speed in the same direction. Now, if the higher plane always remains directly above the lower one, wouldn't you say that both planes had the same speed relative to the ground?

The speed of an airplane with respect to the ground is determined only by its horizontal displacement per unit time. This means that if it travels horizontally a distance of 200 km during each hour of its flight its ground speed is 200 km/hr regardless of its altitude.

Please return to page 128 and select another answer.

YOUR ANSWER --- B

You are correct. For a total vector magnitude of 96 km, an 8 cm line segment showing this vector must have a scale value of 12 km per centimeter, scaled as 1 cm = 12 km.

As a further check on your understanding suppose you are asked to draw a line segment representing a displacement toward the south of 35 km drawn to the scale: 1 cm = 5 km.

Refer to Figure 11. If each small division is 1 centimeter which of the three drawings correctly represents the specified displacement?

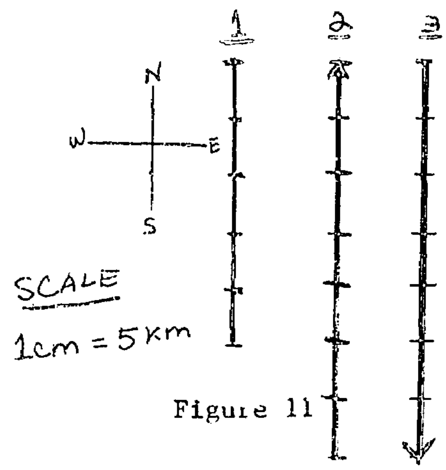


Figure 11

(5)

A 3 is correct.

B 2 is correct.

C 1 is correct.

YOUR ANSWER --- A

What happened to the scale? You counted the number of centimeters in the BC line segment without converting to meters by means of the scale. When you describe the <u>actual</u> displacement, you can't talk in terms of the scale vector itself. You must find the actual distance by using the scale!

Please return to page 32 and select another answer.

YOUR ANSWER --- A

You are correct. If you selected this answer on the first try, your notebook is well kept. Keep it up!

We'd like to point out some shortcuts you can take when vectors at right angles happen to form special kinds of triangles after vector addition. Refer to Figure 14.

In one particular special triangle the legs are in the ratio 3:4. The hypotenuse, then, has an equivalent value of 5. This is the familiar 3:4:5 triangle.

As an example, suppose an airplane travels 18 km due east from X to Y, then 24 km due north from Y to Z. Noting that 18 is to 24 as 3 is to 4, you recognize that you are dealing with a 3:4:5 triangle, so that the total displacement (XZ) of the airplane must be 30 km. That is, 3:4:5 = 18:24:30.

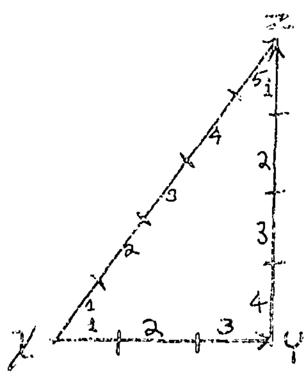


Figure 14

Suppose the legs of the right triangle were, respectively, 24 km and 32 km. What would then be the magnitude of the displacement.

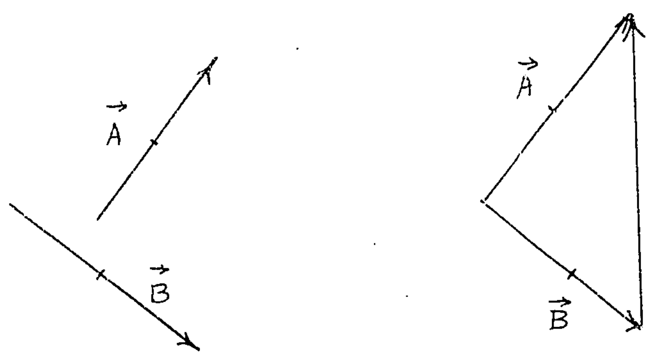
(11)

A The displacement would be 40 km.

B I don't understand how to determine the displacement.

YOUR ANSWER --- A

This answer is false. Refer again to diagram (1) and note especially the direction of the vectors with respect to each other. Now refer to diagram (2). You will observe that, although $\vec{B}$ was shifted to a new position, it was not reversed in direction but connected tail-to-tail with vector $\vec{A}$. If this diagram were supposed to show vector addition, how should the vectors be connected?



(1) (2)

Figure 38

Please turn to page 41 to check your answer.

YOUR ANSWER --- C

You are correct. As in algebra, angles are measured counterclockwise from the X-axis in the 1st quadrant, so that 0^{o}---90^{o} appear in the 1st quadrant. It is consistent, therefore, to refer all angle measurements to a horizontal axis, like the east-west axis in our example, always measuring counterclockwise. Thus, the vector in Figure 6 may also be described as making an angle of 45^{o} to the X-axis. So, you see, vector directions may be specified with reference to points of a compass or by means of standard angle notation. Vector magnitudes may be indicated by the simple device of labeling them with the appropriate figure, or they may be indicated in the form of scaled drawings.

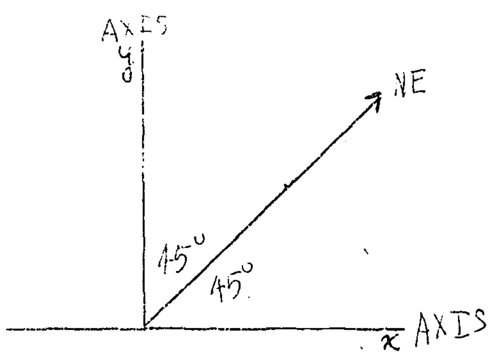


Figure 6

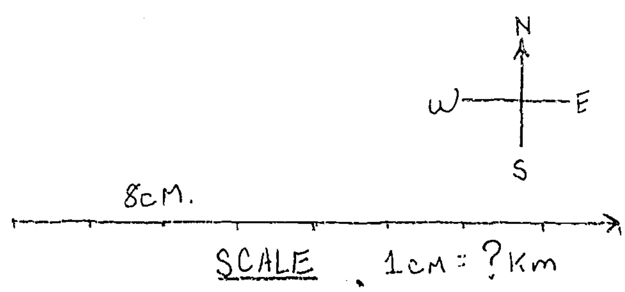


Figure 8

Refer to Figure 8. In this illustration, we wanted to represent a 96 km displacement to the east. We drew a line segment of 8 cm from west to east as shown. Now, in order that anyone looking at this drawing can determine how large displacement is, we must write in a scale, or proportionality constant.

How many kilometers does each centimeter of this line segment represent?

(4)

A 8 km.

B 12 km.

C I don't know.

YOUR ANSWER --- B

You are correct. The motion due east is shown as a vector 8 cm long. Since each centimeter represents 15 meters, then the actual walk east is:

15 m/cm x 8 cm = 120 m

See Sheet

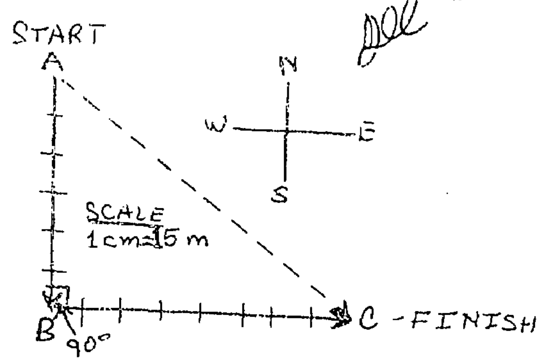


Figure 12

Now we ask "What is the displacement of the boy from his initial position?" He went from A to C via the paths AB, then BC; but he might have taken the path AC in the first place. Thus, his displacement is shown by the length and direction of vector AC. Line segment AC has been measured off in centimeters as you can see.

Using the same scale, what is the magnitude of his displacement from his original position at the end of his walk?

(8)

A His total displacement is 120 m.

B His total displacement is 150 m.

C I can't figure this out.

CORRECT DRAWING:

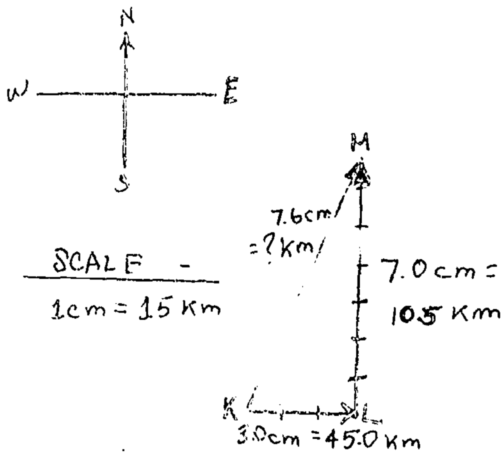


Figure 13

Your drawing should appear the same as Figure 13. The first trip, 45.0 km toward the east, is represented as $\overrightarrow{KL}$ on our drawing, a length of 3.0 cm according to the scale. The second trip, 105 km toward the north, is represented as $\overrightarrow{LM}$ on our drawing, a length of 7.0 cm. By vector addition:

$$\overrightarrow{KL} + \overrightarrow{LM} = \overrightarrow{KM}$$

Hence the displacement of the car is $\overrightarrow{KM}$.

Now measure $\overrightarrow{KM}$ to the nearest millimeter (0.1 cm). If your measurements have been precise, you will find that $\overrightarrow{KM}$ is 7.6 cm long. Therefore, according to the scale, what is the displacement of the car to the proper number of significant figures?

(9)

A The displacement of the car is 114 km along $\overrightarrow{KM}$.

B The displacement of the car is 110 km along $\overrightarrow{KM}$.

YOUR ANSWER --- C

You are correct. That should have been easy for you.

To get the answer, you merely subtracted: $\vec{A} - \vec{B}$. Concerning the <u>magnitude</u> of the difference, this turned out to be 5 mi - 3 mi = 2 mi. The fact that point (3) is <u>south</u> of point (2) is self-evident from the diagram.

This self-evidence, unfortunately, is often lacking in more complicated problems, so we need a <u>formal</u> procedure for finding vector differences. We can get our clue from a simple algebraic fact that everyone knows.

Isn't it true that 9 - 5 is exactly the same thing as 9 + (-5)? That is, if we want the difference between two numbers, we can reverse the sign of the subtrahend (the number that is being subtracted) and then <u>add</u> algebraically. The identical rule applies to vectors. Thus, if we want the difference $\vec{A} - \vec{B}$, we can reverse the sign of $\vec{B}$ and add algebraically like this: $\vec{A} + (-\vec{B})$. Of course a vector whose sign is reversed, will have to be turned so that it points in the opposite direction.

Please go on to page 106.

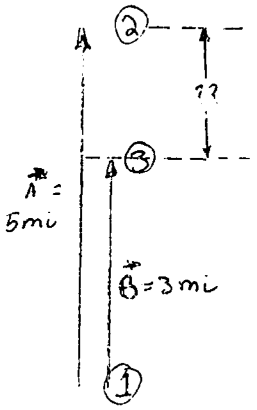


Figure 27

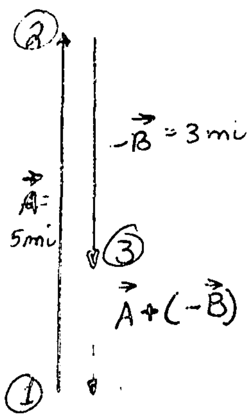


Figure 28

Figure 27 shows the original trips side by side to remind you that we are dealing with a single road. In Figure 28, $\vec{B}$ has had its direction reversed, and has been shifted parallel to itself so that its tail is joined to the head of $\vec{A}$. Following the method of vector addition, (now that $\vec{B}$ has been converted to $-\vec{B}$), the diagram is completed by drawing in the dotted line segment from (3) to (1). We have made this dotted only to emphasize it.

What is the magnitude and direction of the difference vector labeled $\vec{A} + (-\vec{B})$?

Please turn to page 135 to check your answer.

YOUR ANSWER --- C

Not a good choice of scale. With this scale, the vector diagram would consist of a 4-cm line segment (the plane's velocity) at right angles to a 1-cm line segment (the wind's velocity). This makes up a very small diagram and tends to reduce accuracy. We would not call it wrong but we certainly would not call this the best choice of scale.

Please return to page 89 and see if you can't find a better scale.

YOUR ANSWER --- C

What difference does it make? Vector quantities are handled in the same way whether they represent velocities, displacements, or any other quantities. This is the value of a general analysis such as the one we are now engaged in. We develop general principles and techniques, and then apply them to specific problems.

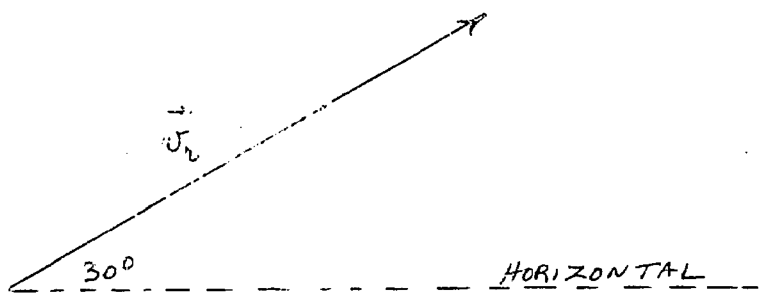


Figure 44

For a general approach, you don't need to know what particular velocity $\vec{v}_r$ represents.

Please return to page 133 and select another answer.

YOUR ANSWER --- A

Not true. In the actual situation, AB merely represents the "heading" of the swimmer. The river current prevents him from actually taking this path. Furthermore, vector $\vec{AB}$ certainly is not the direction and magnitude of the water velocity, $\vec{v}_w$.

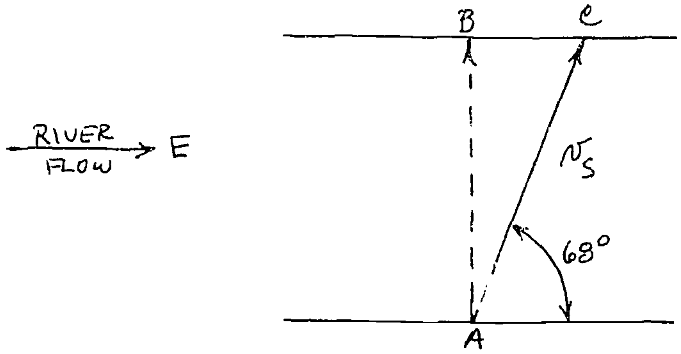


Figure 47

Please return to page 93 and select another answer.

YOUR ANSWER --- B

Incorrect. You were asked to calculate (or measure) $\vec{v}_w$ to 2 significant figures. Your answer does not contain 2 significant figures. You can get closer than this.

Please return to page 95 and select another answer.

YOUR ANSWER --- B

Somehow, you set up the ratio improperly. Here's the right way to do it.

$$\frac{20}{5} = 4 \qquad \text{and} \qquad \frac{48}{12} = 4$$

Therefore:

$$\frac{20}{48} = \frac{5}{12}$$

The legs are in the ratio 5:12. So the ratio of all sides of the triangle must be 5:12:13 for this special case. To determine the hypotenuse, now, we need simply multiply 13, the last term of the ratio by 4, the common multiplier. This does not come out 56, does it?

Please return to page 14 and select another answer.

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YOUR ANSWER --- B

Your answer is incorrect.

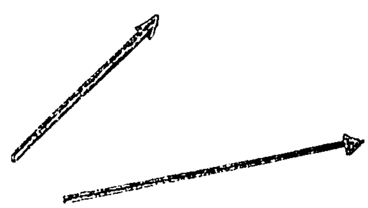

Figure 24

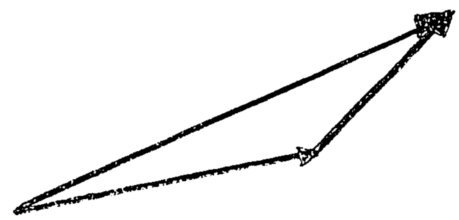

Figure 25

Although it is quite true that both vectors have been moved, this does not make the finished diagram incorrect. It's true that in discussing the parallel-movement principles, we spoke of moving only one of the vectors, but this was intended as a matter of convenience rather than as a requirement. In other words, most vector problems can be handled easily by shifting just one of the vectors. But, if necessary, both vectors may be moved without violating any rules, provided that the shift is accomplished without changing the direction or magnitude of either line segment.

Please return to page 15 and select another answer.

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YOUR ANSWER --- D

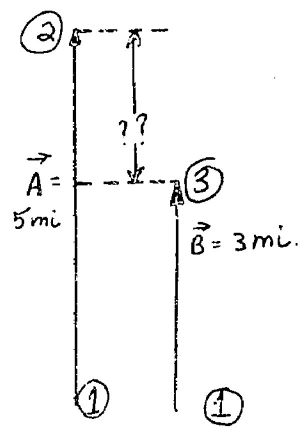

Figure 27

We have brought the original vectors closer to each other in Figure 27. Perhaps this will make it easier for you to see that the magnitude of the difference between the position of point (3) relative to point (2) is not 3 miles.

You were right in one respect: relative to point (2), point (3) is south of the more distant position.

Please return to page 126. Check back on the magnitude of the difference and choose a better answer.

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YOUR ANSWER --- A

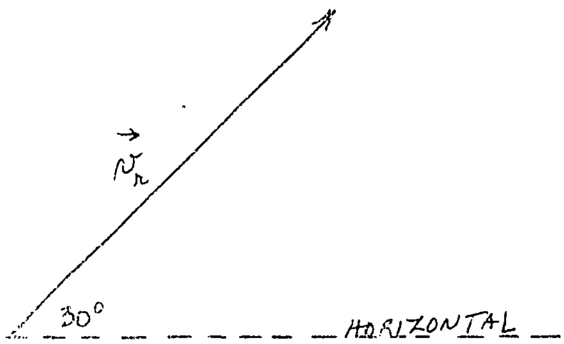
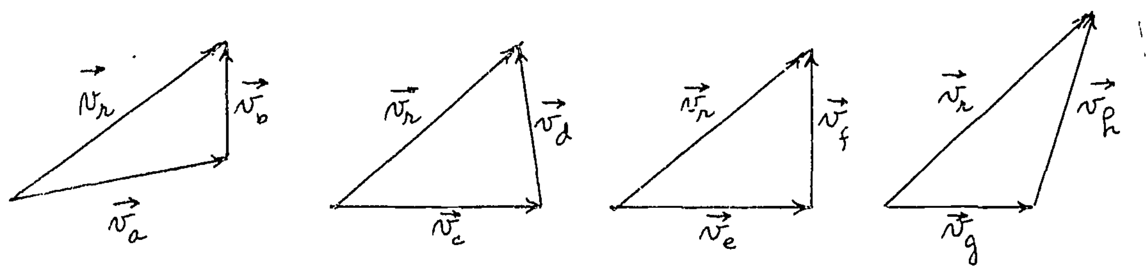


Figure 44

You are correct. The velocity $\vec{v}_r$ has an infinite number of possible pairs of components, hence you must be informed which particular pair is desired.

Let's specify the desired pair. We'll reword the question: "Resolve $\vec{v}_r$ into its vertical and horizontal components." Have you enough information now? Yes, indeed, you have!

A horizontal line is perpendicular to a vertical line. This means that the two components must be at right angles to each other. Also the words horizontal and vertical have very special significance. A horizontal line is parallel to the bottom of your paper and a vertical line is parallel to either side of your paper. Now, only one of the diagrams below shows $\vec{v}_r$ correctly resolved into vertical and horizontal components. Which one is it?



(1) (2) (3) (4)

Figure 45

(31)

A (3) is correct.

B (4) is correct.

C (1) is correct.

D (2) is correct.

YOUR ANSWER --- B

You are correct. The displacement vector measures 13 cm. Applying the scale, we may write:

$$13 \text{ cm} \times 0.5 \text{ mi/cm} = \underline{6.5} \ \underline{\text{mi}}$$

Our work up to this point has been the addition of vectors representing individual trips to determine the resulting displacement. We chose this topic to start our study of vectors because it doesn't require too much imagination to see why we drew the two vectors head-to-tail. The second trip naturally begins where the first leaves off.

Unfortunately, this situation does not always hold true. Vectors are employed in many, many phases of physics where the head-to-tail orientation is difficult to visualize. We can make our understanding of vectors more general and, therefore, more useful by considering how to handle vector addition when the vectors represent quantities other than displacements.

Please go on to page 119.

In Figure 18 we have drawn two vectors, $\vec{A}$ and $\vec{B}$. We will not be concerned with the quantities they represent and, in fact, we will assume that we don't know what they stand for. We want to obtain their sum. How do we go about it?

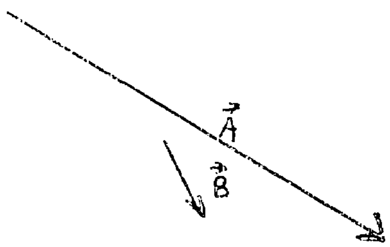


Figure 18

The sum of any two vectors may be found, regardless of where they start, by moving either one of the vectors parallel to itself until its tail can be joined to the head of the other vector. This movement is shown in Figure 19. Vector $\vec{B}$ has been moved to the right, and slightly downward, parallel to its original position, so that its tail joins the head of $\vec{A}$. The sum of the two vectors is then the third leg of the triangle and is identified as $\vec{A} + \vec{B}$.

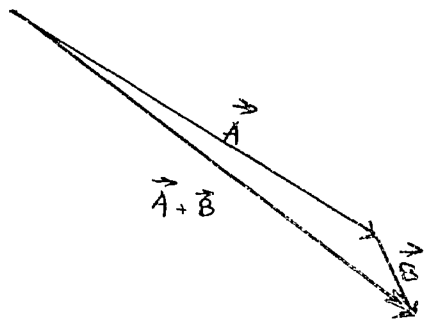


Figure 19

Please go on to page 120.

In accordance with the principle just outlined, inspect the vectors in Figure 20; then choose the one diagram below that you consider to be a correct application of the principle.

(14)

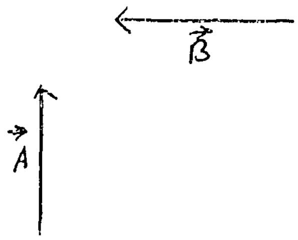


Figure 20

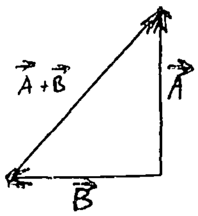


Figure 21

A - This diagram is correct.

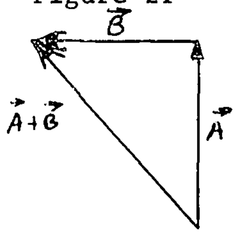


Figure 22

B - This diagram is correct.

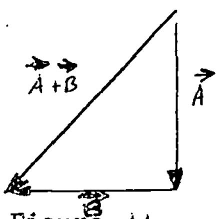


Figure 23

C - This diagram is correct.

YOUR ANSWER --- A

No, that's incorrect. The direction "northeast" is defined as 45° east of north. Note that the vector has been drawn exactly 45° to the east of the due north direction; hence its direction may be defined as precisely northeast. As you will discover, whenever possible our measurements and descriptions of physical objects or events are made precise rather than approximate. There is no reason why an exact statement of direction cannot be made in this case. To give it approximately would be to ignore the facts presented.

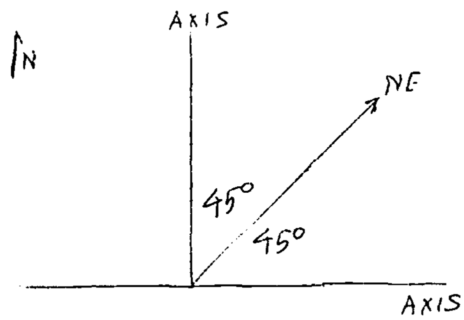


Figure 6

Please return to page 13 and select another answer.

YOUR ANSWER --- B

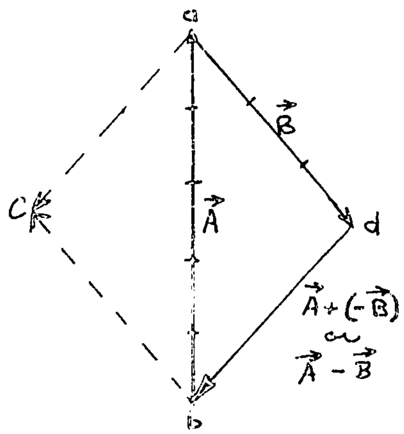


Figure 33

You are correct. $\vec{B}$ has been reversed and has been shifted to the top of $\vec{A}$ so that the two vectors are joined head-to-tail.

Figure 33 is a kind of composite diagram which shows that the first "short cut" solution we used and the correct procedural solution yield identical answers. The original $\vec{A}$ is now identified as line segment ba. You will recall that line segment ac was the "short-cut" answer to the difference problem. Here we joined the vectors tail-to-tail. The answer obtained by rigidly adhering to the rules is line segment db. If ac and db have the same magnitude and direction, then either method will give the right answer. You notice that we are not interested in the point of application as shown, only in direction and magnitude.

Which of the following theorems may be used here to prove that ac and db have equal magnitude and direction?

(20)

A $\triangle$acb is similar to $\triangle$adb. Similar triangles have proportional sides.

B The opposite sides of any polygon are equal and parallel to each other; hence ac and db are equal in magnitude and have the same direction.

C Polygon adbc is a parallelogram since ad was made equal and parallel to cb by construction.

YOUR ANSWER --- C

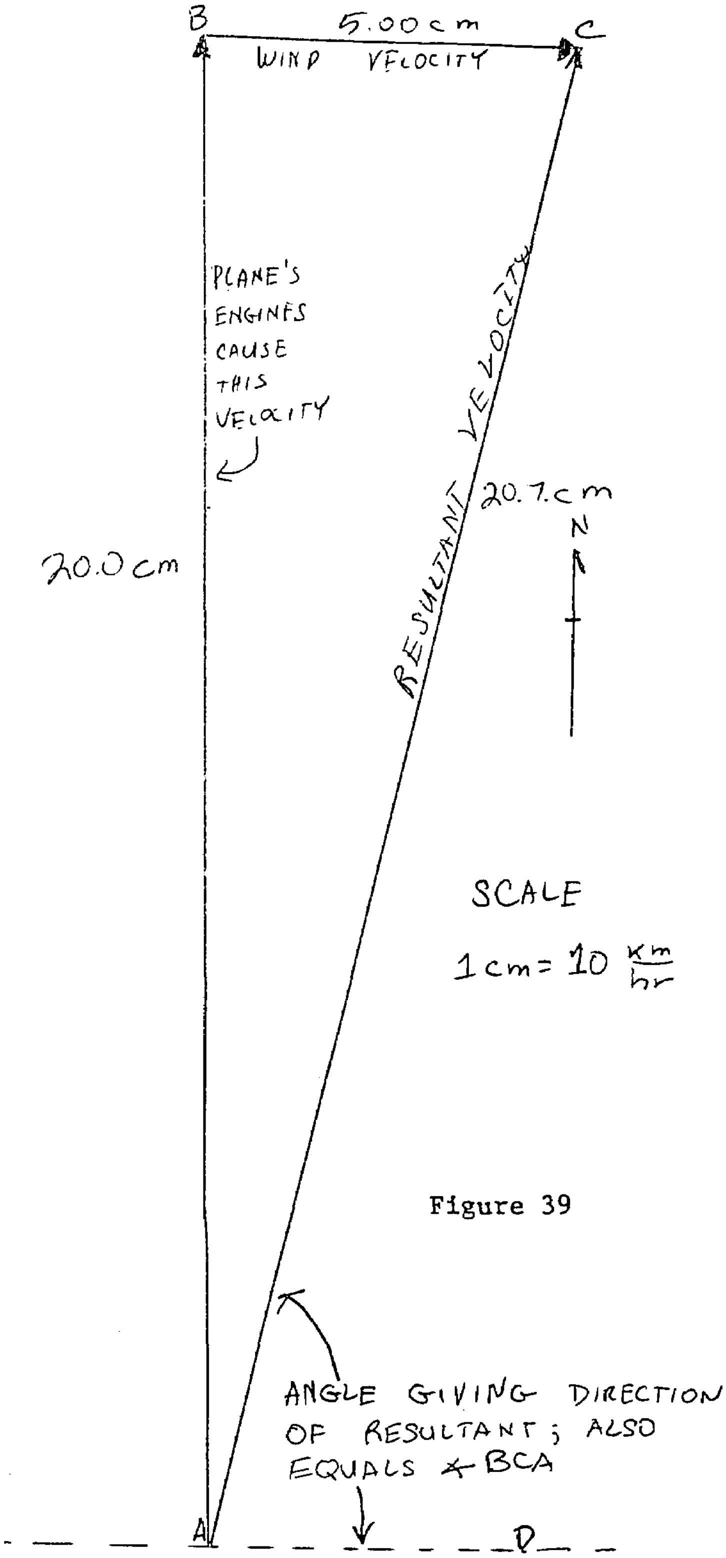


Figure 39

You are absolutely correct. The resultant turns out to be 20.7 cm long which, in terms of the scale, is 207 km/hr. Angle CAD (or BCA) is 76°.

Try another velocity problem on your own. (Draw a picture of the situation on scrap paper to help you visualize it.)

Two points, A and B, are located directly opposite each other on the banks of a river. The river, which flows from north to south, has a velocity of 17.5 km.hr. If a boat heads due west (from B to A), what will be its resultant velocity if its engine can move it forward at a speed of 44.0 km/hr? State the resultant speed to 3 significant figures and the direction in terms of the angle between the resultant vector and an east-west line to the nearest degree.

To check your answer, please turn to page 92.

YOUR ANSWER --- D

This is probably a careless mistake. We are not saying that the numerical part of your answer is right, but we must say that your directions are mixed up. Could BC possibly be directed toward the west? Your diagram doesn't seem to be accurate. Better sharpen your pencil.

Please return to page 95 and select another answer.

YOUR ANSWER --- C

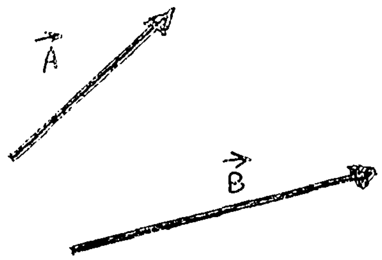


Figure 24

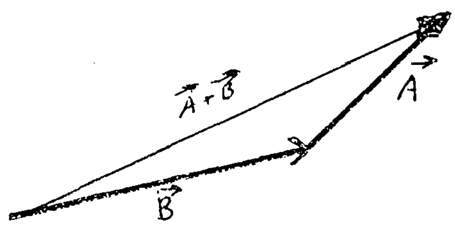


Figure 25

You are correct. Fine! The rules have all been carefully observed: (1) the vector shift has taken place parallel to the initial positions; (2) there has been no alterations of magnitude; (3) the final vectors are joined head-to-tail as they should be.

Now we know how to find vector sums. We know that when two vectors are added by the method of vector addition using directed line segments, completion of the triangle provides us with the magnitude and direction of the vector sum.

Is there some corresponding method for finding vector difference? If there is, we had better find out beforehand why we should ever want to subtract one vector from another.

Many physical processes and phenomena involve vector differences, but we shall ask you to be satisfied with only one example at this time, since you are not sufficiently far advanced to comprehend most of the others.

Please go on to page 126.

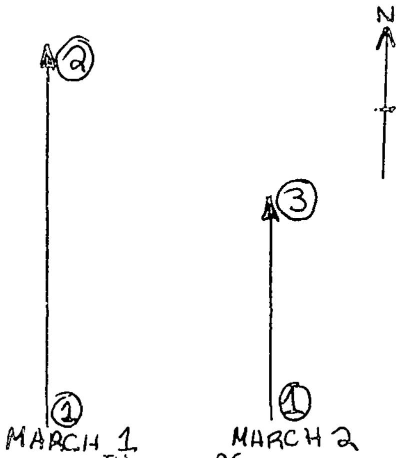


Figure 26

Let's see what story is told by the vector diagram in Figure 26. On March 1 a car is driven from point (1) to point (2), a distance of 5 miles as represented by $\vec{A}$. On March 2, the same car is driven from the same point (1) to a different point along the same straight road; this other point is identified as (3) in the diagram and is only 3 miles from point (1), as shown by $\vec{B}$. Now, remember that we are speaking of the same road and that we have displaced the vectors sideways only to keep the diagram clear.

Using nothing but common sense, you should now be able to answer this question: What is the magnitude of the difference between the position of point (3) relative to point (2), and in what direction does point (3) lie relative to point (2)?

(16)

A Point (3) is 2 mi north of point (2).

B Point (3) is 3 mi north of point (2).

C Point (3) is 2 mi south of point (2).

D Point (3) is 3 mi south of point (2).

YOUR ANSWER --- B

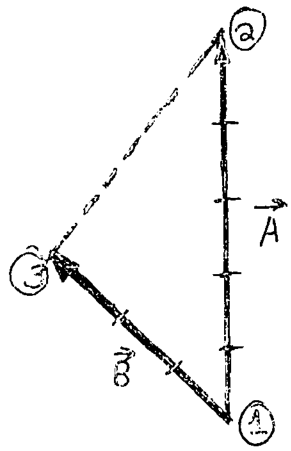


Figure 29

You are correct. The rule states that for either addition or subtraction, vectors must be connected head-to-tail. In our drawing, $\vec{A}$ and $\vec{B}$ are connected tail-to-tail.

Well, then, as obvious as the answer may be, we cannot justify violating a rule to obtain it. Of course, as you shall see, exactly the same answer can be obtained using standard procedure as given in notebook entry (3).

Item (3) tells us to change the sign of the subtrahend by reversing the direction of $\vec{B}$, and then to shift $\vec{B}$ until it is joined head-to-tail with $\vec{A}$.

Which one of the following diagrams shows this rule properly executed?

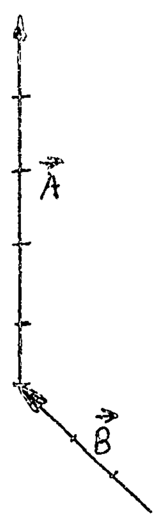


Figure 30

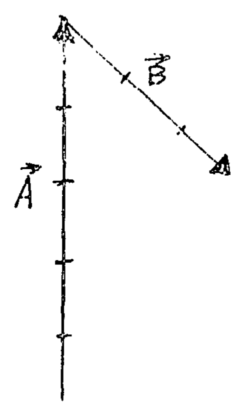


Figure 31

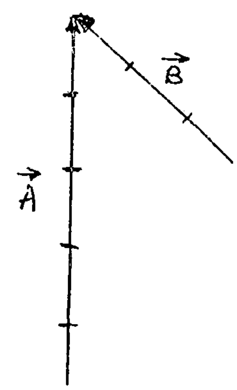


Figure 32

(19)

A Figure 30.

B Figure 31.

C Figure 32.

YOUR ANSWER --- C

You are correct. Very good. Diagram (2) illustrates the short-cut method of subtracting vectors. Diagram (3) shows the same result obtained by the standard method.

If displacements were the only vector quantities with which the physicist must deal, this lesson would end here, now that you have learned how to add and subtract displacements. But you must now learn how to apply the same techniques to other vector quantities.

Before continuing, please turn to page 143 in the blue appendix.

Suppose an airplane has a speed of 200 kilometers per hour (km per hr or km/hr) in the air. Imagine at the same time that the air is moving with a wind speed of 50 km/hr. How fast, then, is the airplane moving relative to the ground? Obviously, the engines of the airplane move it at a definite velocity with respect to the ground. But so does the wind. These two velocities are independent of one another but both affect the final velocity of the airplane.

The moment you try to answer the question above, you run into trouble. Exactly what is the trouble?

(22)

A We must know the wind direction to solve the problem.

B We must know the altitude of the plane in order to determine its velocity relative to the ground.

C Since the earth turns under the airplane, we would have to take this motion into account.

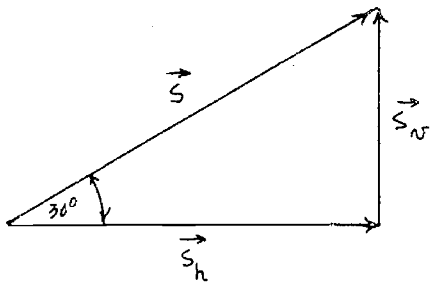


Figure 46

From the tip of $\vec{s}$, a perpendicular line segment was constructed on the ground level line. (This may be done either with a protractor or a compass.) Now, you must draw the diagram to scale as we have it thus far. Clearly, $\vec{s}_v$ will then give the height of the top of the slope from the ground and $\vec{s}_h$ will give the length of the horizontal component of the trip.

Now, using your scale, determine the answers to the two questions:
(a) How far did he travel horizontally?
(b) How high is the top of the slope above ground level?

Determine both answers to 3 significant figures and write them down.

Please turn to page 93 to check your answer.

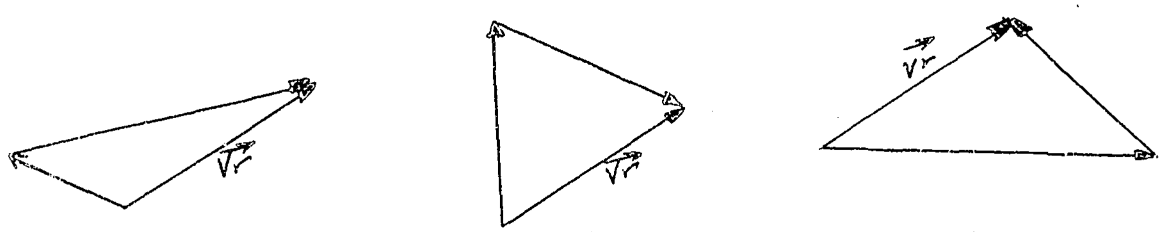
YOUR ANSWER --- B

Incorrect! Item 3(c) reads: Thus, to subtract $\vec{B}$ from $\vec{A}$, reverse the direction of $\vec{B}$ and join it to $\vec{A}$ in the usual head-to-tail manner.

Please return to page 17. Let's hope your next try is better!

YOUR ANSWER --- B

Not at all! Look at diagram Figure 43.



(A) (B) (C)

Figure 43

Here are three additional pairs of possible components for the same resultant $\vec{v}_r$. These are just as good as any of the other pairs previously illustrated.

Why should the number of possible pairs of components for a given resultant be limited to four, or ten, or 1,000?

Please return to page 25 and select another answer.

YOUR ANSWER --- C

Let's try a different approach. Suppose we start out by saying that we are going to draw an eastward vector representing a certain number of kilometers. Suppose we choose a scale like this: 1 cm = 3 km. Next, we draw a 1-cm line like the one in Figure 9. At this point you would have no trouble finding that the total magnitude of this vector is 3 km, since it is 1 cm long and each centimeter represents 3 km.

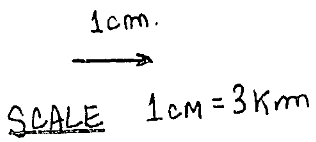


Figure 9

Right. Now refer to Figure 10. How many kilometers are represented by this vector 2 cm in length? Remember, each centimeter represents 3 km.

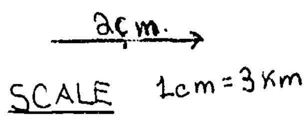


Figure 10

Please turn to page 42 to check your answer.

YOUR ANSWER --- C

You are perfectly correct. You saw four possible pairs of components in Figure 42. Here are three other possible pairs.

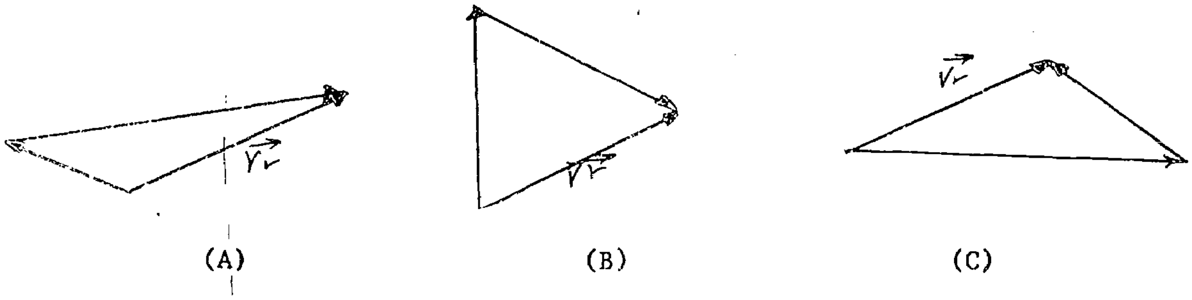


Figure 43

Thus, you can see that the number of pairs of possible components has absolutely no limit.

When a given vector is broken down into a specific pair of components, we say that we have resolved the original vector into components. The process itself is called resolution of vectors.

Since any vector quantity may be resolved into an infinite number of possible pairs of components, if you are asked to perform a resolution on a given velocity, say, you will have to be supplied with additional information. Assume, for example, that you have been asked to resolve the velocity $\vec{v}_r$ in Figure 44 into its two component velocities. Recognizing that the information you have been given is inadequate, which one of the following questions would you ask?

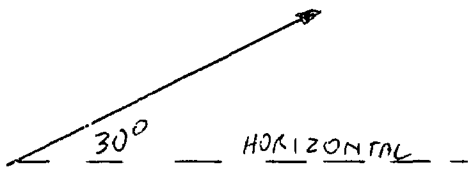


Figure 44

(30)

A Which components specifically are wanted?

B What is the magnitude of $\vec{v}_r$?

C Is $\vec{v}_r$ the velocity of a boat or stream?

YOUR ANSWER --- B

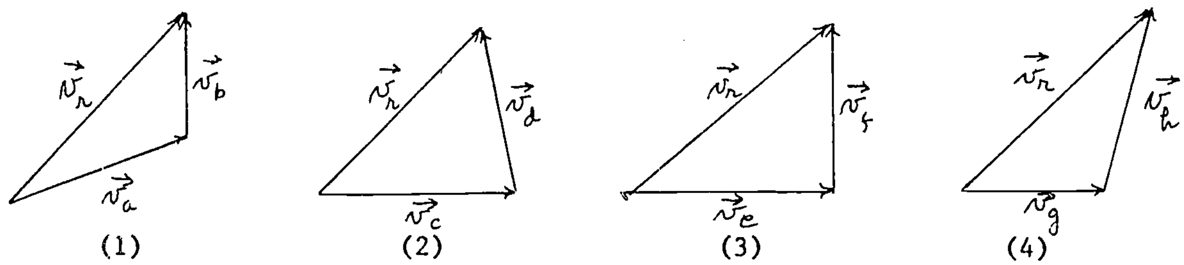


Figure 45

Incorrect. In (4), $\vec{v}_g$ is horizontal but $\vec{v}_h$ is <u>not</u> vertical. The conditions of the problem call for a right angle between $\vec{v}_g$ and $\vec{v}_h$. The angle, as you can see, is more than 90°. <u>This makes both $\vec{v}_g$ and $\vec{v}_h$ incorrect components</u>.

You might think that $\vec{v}_g$ is the correct horizontal component merely because it is a horizontal line segment. This is not true because, should $\vec{v}_h$ be rotated until it makes a right angle with $\vec{v}_g$, then either the resultant or $\vec{v}_g$ would have to change in length.

Thus, (4) does not represent $\vec{v}_r$ resolved into its vertical and horizontal components.

Please return to page 117 and select another answer.

CORRECT ANSWER: The magnitude of the difference vector is 2 mi. Its direction is south of point (2). If we had joined the head of $-\vec{B}$ to the tail of $\vec{A}$, the point of application of the difference vector would be more obvious.

Let us review this solution. A car travels 5 mi north along a road from a certain point. Later, it goes only 3 mi north along the same road from the same starting point. What is the difference in the car's position after the second trip relative to its finishing position after the first trip? That answer is, of course, that after the second trip the car is 2 mi south of the previous finishing point.

The notebook entry below summarizes the procedure for finding the difference between two vectors. Although we used the simplest possible case, that is, two vectors that lie along the same line, the procedure is equally applicable to vectors at any angle, as you will see.

NOTEBOOK ENTRY

3. Vector Subtraction

(a) The difference between two vectors, expressed as $\vec{A} - \vec{B}$, is best obtained by changing the sign of the subtrahend and writing:
$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

(b) When the sign of a vector is changed from + to −, the direction of the vector must be reversed.

(c) Thus, to subtract $\vec{B}$ from $\vec{A}$, reverse the direction of $\vec{B}$ and join it to $\vec{A}$ in the usual head-to-tail manner.

(d) The vector difference is then given by the magnitude and direction of the line segment joining the head of $\vec{B}$ to the tail of $\vec{A}$.

If $\vec{U}$ is to be subtracted from $\vec{V}$, in which of the following forms should our vector notation be written?

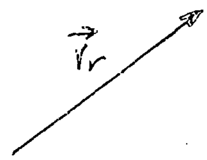
(17)

A $\vec{U} - \vec{V}$

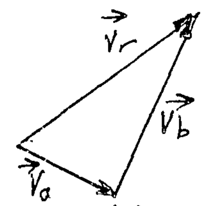
B $\vec{V} - \vec{U}$

C $\vec{V} + (-\vec{U})$

YOUR ANSWER --- A



(A)



(B)

Figure 41

If $\vec{v}_r$ is to be the resultant of $\vec{v}_a$ and $\vec{v}_b$, then the two components must be added in the proper manner to obtain the sum vector. Let's see whether or not this addition has been performed correctly.

In order to add a pair of vectors, all you have to do is to join them head-to-tail without allowing their initial directions or magnitudes to change. The velocity $\vec{v}_a$ is one vector; the velocity $\vec{v}_b$ is the second vector; the head of $\vec{v}_a$ is joined to the tail of $\vec{v}_b$. Then the vector $\vec{v}_r$ is used to complete the triangle, an arrowhead being added to it to show the final direction of this velocity. Thus, $\vec{v}_r$ is truly the vector sum of $\vec{v}_a$ and $\vec{v}_b$.

According to our definitions of components and resultants, the resultant is obtained from the vector sum of the components. Since $\vec{v}_r = \vec{v}_a + \vec{v}_b$, then $\vec{v}_r$ is definitely the resultant of the two components $\vec{v}_a$ and $\vec{v}_b$.

Please return to page 92 and select the right answer.

YOUR ANSWER --- D

This choice has two strikes against it.

First, never start your scale with anything but 1 cm. When you start with some peculiar choice like 5 cm = something-or-other, you immediately begin to complicate your final arithmetic. Again, this isn't wrong, but it is very inconvenient. Glance at the scale of miles on any good map. You will find that the scale always starts with 1 inch = xxxxx or 1 cm = xxxxx.

Second, if you reduce this scale to 1 cm = 1 km/hr, which is the same thing, your drawing would consist of a 50-cm line segment (the wind's velocity) at right angles to a 200-cm line segment (the plane's velocity). A line segment 200 cm long is 2 meters long, or over 2 yards! You'd need quite a monstrous sheet of paper for this job.

Care must be exercised in the choice of a scale to be certain that the final drawing will be neither too small nor too large.

Please return to page 89 and select another answer.

YOUR ANSWER --- D

Not a good guess. Rule 2(a) reads: The displacement resulting from two trips in the same direction along a straight line is the algebraic sum of the two trip lengths. Arbitrary signs are given to each trip. (+) in both cases since they are in the same direction.

Please return to page 79 and select another answer.

YOUR ANSWER --- A

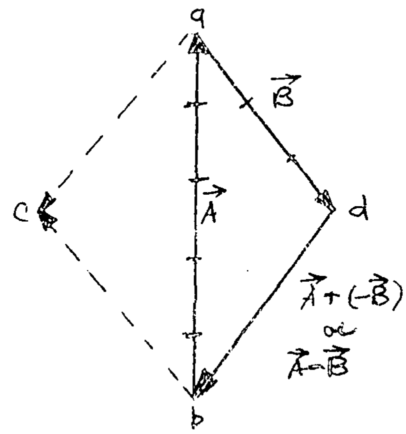


Figure 33

This is incorrect. The triangles do happen to be similar and the sides proportional, but we are not interested in proportionalities here. We have to show that corresponding sides are equal and parallel. So this theorem is not suitable.

You might show that $\triangle$ abc is congruent to $\triangle$ adb. This is, as a matter of fact, quite easy to do. However, this was not one of the choices you were given.

Please return to page 122 and select another answer.

This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

WORKSHEET

Please listen carefully to tape segment 1 for this lesson before starting this Worksheet.

Your answer selections for the questions that appear below are to be punched out on the special AV Computer Card for this lesson.

1. When a scalar quantity is multiplied by a vector quantity, the product is always a vector quantity. Which one of the following products is a vector quantity?

   A time x temperature.
   B mass x time.
   C mass x velocity.
   D mass x temperature.
   E none of the above is a vector quantity.

2. If you want to know the displacement of a given body traveling with a given speed, then you must also know

   A the magnitude of the body's speed.
   B the magnitude of the body's velocity.
   C the time allowed for the trip.
   D the distance between the two points.
   E the time and direction of the motion.

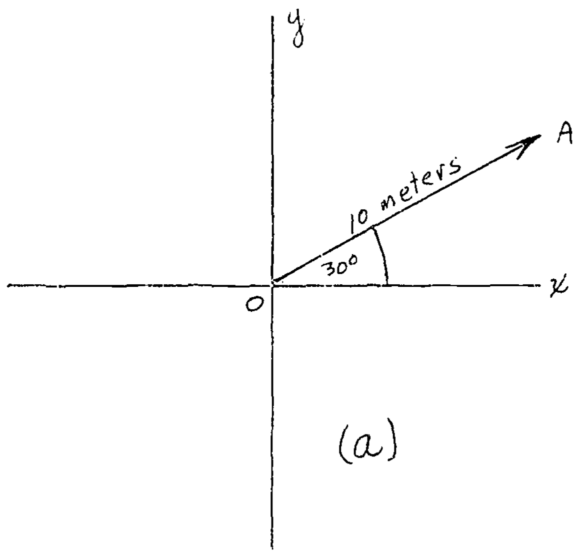
3. Weight may be defined as the pull of gravity on a material body near the surface of the earth, the pull being directed toward the center of the earth. On the basis of the discussion in tape segment 1 and this definition, which one of the following statements is the true one?

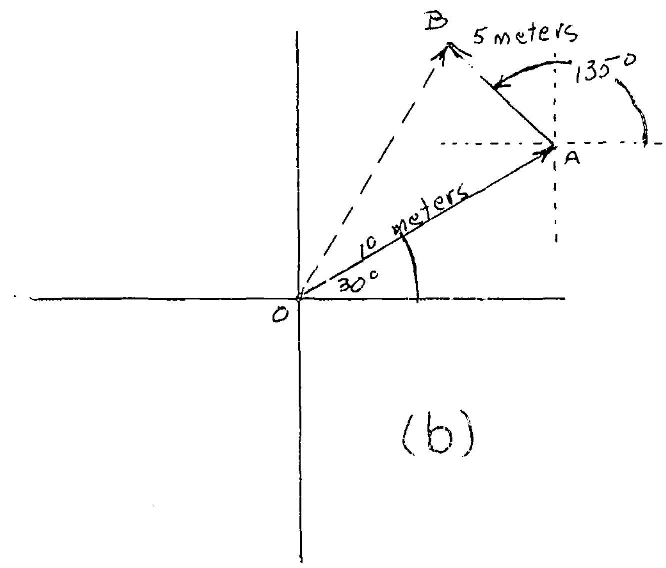
   A Weight is a vector quantity.
   B Weight is a scalar quantity.
   C Weight may be considered either as a vector or a scalar quantity depending on the circumstances.
   D It is not possible to determine whether weight is a vector or a scalar quantity with the information given above.
   E Weight is neither a scaler nor a vector; it is a special kind of quantity to be discussed later.

(Please return to page 5 of the STUDY GUIDE
to continue with the lesson.)

WORKSHEET

Please listen carefully to tape segment 2 for this lesson before starting this Worksheet.





QUESTIONS

4. At what angle to the x-axis would the man have had to walk on his second trip in order that the total displacement after both trips be 5 meters at 30° to the x-axis?

   A 180°.

   B 210°.

   C 30°.

   D 105°.

   E 165°.

5. Suppose, on the second trip, the man had walked a distance of 7 meters at an angle of 30° to the x-axis. His net displacement at the end would then be

   A 17 m at 30°.

   B 3 m at 30°.

   C 17 m at 135°.

   D 3 m at 135°.

   E none of these is correct.

(Please continue to the next page).

WORKSHEET

1. Draw a set of rectangular axes at the center of a clean sheet of notebook paper using a very sharp, moderately hard pencil. Use your centimeter rule and protractor to help.

Using a scale of 1 cm = 1 meter, show three trips in sequence starting from the origin according to the following data:

TRIP 1: 5 m at 233°
TRIP 2: 8 m at 90°
TRIP 3: 4 m at 307°

When the diagram is complete, determine by graphical methods the magnitude and angle of the resultant or net displacement, then select the values which are closest to yours from the list below:

A 2 m at 37°.
B 1 m at 37°.
C 1 m at 45°.
D 1 m at 127°.
E 2 m at 127°.

(Please return to page 79 of the STUDY GUIDE to continue with the lesson).

WORKSHEET

[illegible] is no tape segment for this worksheet. Proceed directly to the [illegible] below for additional practice and clarification of the material presented in the STUDY GUIDE.

[illegible] answer selections for Worksheets that are not accompanied by audio tape are also punched out on the same AV Computer Card as those [illegible] other Worksheets.

A car traveling due north from the town of Haines along a straight highway reaches the town of Fowler after covering a distance of [illegible] miles. The next day, another car starting from exactly the same point in Haines journeys 35 degrees north of east a distance of 5.0 miles to arrive at the town of Castle.

[illegible] How far is Fowler from Castle?

A [illegible] miles.
B [illegible] miles.
C 6.0 miles.
D 6.6 miles.
E 6.9 miles.

8. In what direction would a car leaving from Castle have to travel in order to arrive at Fowler?

A 39° west of north.

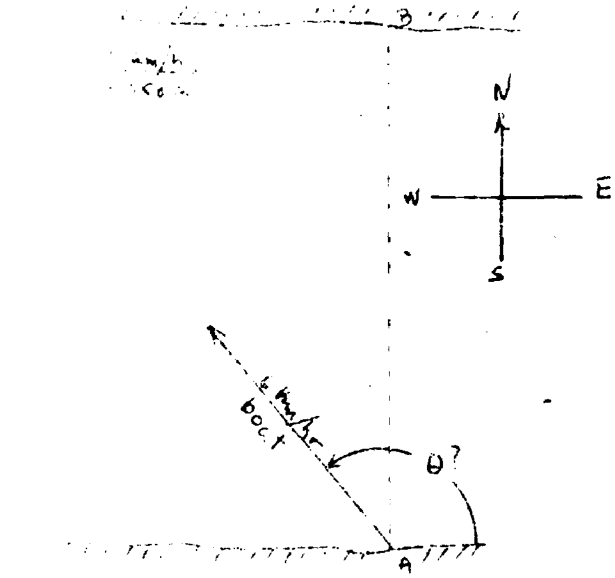
B 45° north of west.

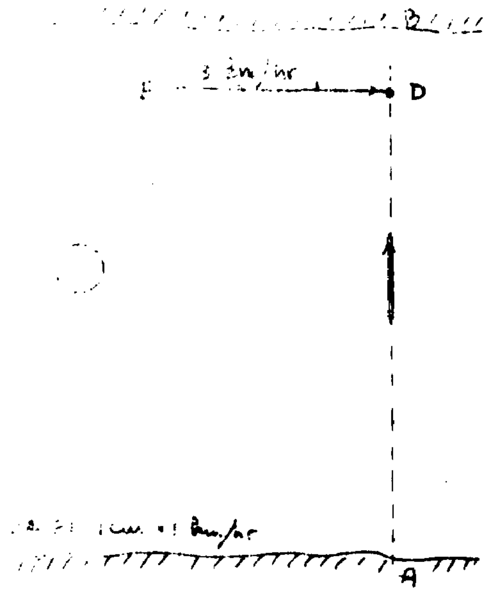
C 89° north of west.

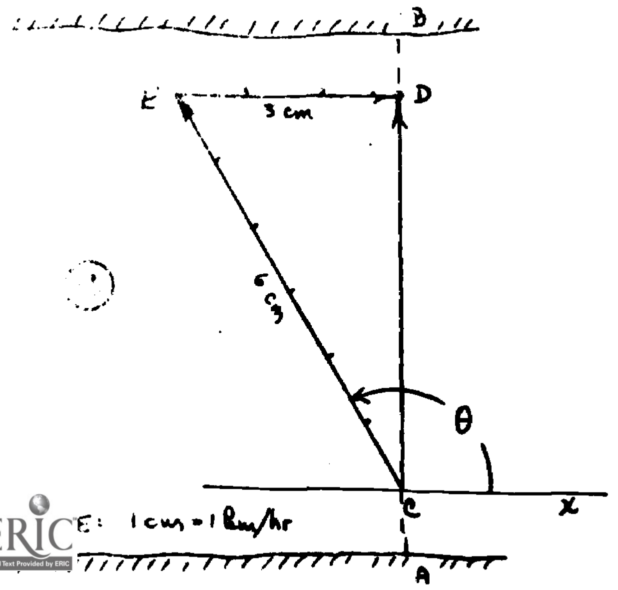
D 35° west of north.

E The direction cannot be determined: insufficient data.

(Please return to page 128 of the STUDY GUIDE to continue with the lesson).







WORKSHEET

Please listen to tape segment 4 before starting this Worksheet.

All answers are to be punched out on the AV Computer Card.

QUESTIONS

12. In Diagram (a), the angle A is

- A 135°.
- B acute.
- C known to be less than 100°.
- D known to be greater than 135°.
- E not yet determined.

13. In Diagram (b), point D

- A must be shown 1 cm from the bank.
- B need not lie on line AB.
- C need not be directly north of point A.
- D must lie on line AB.
- E is specified by the x-y coordinates given in the problem.

14. In the final solution, the boat must head

- A upstream at 120°.
- B upstream at 135°.
- C downstream at 30°.
- D downstream at 60°.
- E upstream at 30°.

15. The resultant velocity of the boat at point D is

- A 6 km/hr, northwest.
- B 5.2 km/hr, northwest.
- C 5.0 km/hr, northwest.
- D 5.2 km/hr, north.
- E 6 km/hr, north.

(Please return to page 27 of the STUDY GUIDE to continue with the lesson).

HOMEWORK PROBLEMS

LESSON 4

1. A 60 mi/hr wind is blowing due north. What is the magnitude and direction of the velocity of an airplane traveling at an air speed of 150 mi/hr when it is heading (a) north; (b) south; (c) east?

2. A boat whose speed with respect to the water is 6 mi/hr is sailing downstream at a 60-degree with the current whose speed is 3 mi/hr. What is the resultant speed and direction of the boat? (Two significant digits for both answers, please).

3. A bullet is fired due east from a gun mounted in an airplane traveling due northeast. If the speed of the bullet is 300 m/sec and the speed of the airplane is 150 m/sec, what is the resultant velocity of the bullet? (Speed and direction, both to 2 significant digits.

4. An airplane heading 45 degrees south of east at an air speed of 120 m/sec is being blown eastward by a 30 m/sec wind. What is the plane's resultant ground speed? (Three significant digits.)

5. An airplane flies 2 000 m north, then 1 500 m due northeast, and finally 3 000 m due south. Determine
   (a) graphically the distance between the airplane's starting and ending positions.
   (b) the resultant direction in which the airplane was displaced.

6. A boat is heading directly across a river at 10 mi/hr. The current is 6 mi/hr and the river is 1.0 mi across.
   (a) How long will it take the boat to cross the river?
   (b) How much further downstream will the boat be arriving at the opposite bank?
   (c) What is the resultant velocity of the boat? (Speed and direction to 3 significant digits)

NOTE: All problems are to be solved on standard 8-1/2 x 11 inch notebook paper and numbered to correspond to the above designations. The solutions must be submitted to your instructor before you may request the Post Test for this lesson. Be sure to enter your name, section, date, and identification number on the submitted work.